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# The Asymptotic Behavior of Conditional Granger Causality with Respect to Sampling Interval

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**Abstract.** Granger causality (GC) stands as a powerful causal inference tool in time series analysis. Typically estimated from time series data with finite sampling rate, the GC value inherently depends on the sampling interval $\tau$. Intuitively, a higher data sampling rate leads to a time series that better approximates the real signal. However, previous studies have shown that the bivariate GC converges to zero linearly as $\tau$ approaches zero, which will lead to mis-inference of causality due to vanishing GC value even in the presence of causality. In this work, by performing mathematical analysis, we show this asymptotic behavior remains valid in the case of conditional GC when applying to a system composed of more than two variables. We validate the analytical result by computing GC value with multiple sampling rates for the simulated data of Hodgkin-Huxley neuronal networks and the experimental data of intracranial EEG signals. Our result demonstrates the hazard of GC inference with high sampling rate, and we propose an accurate inference approach by calculating the ratio of GC to $\tau$ as $\tau$ approaches zero.

**AMS subject classifications**: 62H20, 62F12, 62J05, 62M10

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**Key words**: Causal inference, conditional Granger causality, sampling rate, asymptotic behavior, Hodgkin-Huxley model.

# 1 Introduction

The investigation into the structure and function of the brain has always been an important topic in neuroscience research. Within the pursuit of this field, a crucial aspect is to unravel the functional interactions across different neurons or cortical areas in the brain. With the development of neuroscience techniques, from invasive recordings to noninvasive methods [30, 35], activities can be recorded simultaneously from multiple neurons and across different brain areas. And recent years have seen numerous works about functional connectivity and causal modeling [14, 24] based on brain signals, attempting to provide insights into the patterns of brain functionalities. "Functional connectivity" measures the degree of co-activation of neurons or brain areas, which is often calculated by the correlation coefficient between neural time series data. However, correlation coefficient cannot provide directional information about the interactions.

Granger causality (GC), proposed in 1950s, has been introduced to the field of neuroscience since the beginning of this century [5, 6, 11, 12, 37]. GC can infer the direction of causal interactions between neural signals, and has been applied to various types of neural data, including single-cell and multi-cell recordings [4], local field potentials (LFPs) [9,28], electroencephalography (EEG) [17,18], magnetoencephalography (MEG) [13], and functional magnetic resonance imaging (fMRI) [7, 31, 41]. In recent years, there has been a gradual emergence of research focusing on the theoretical basis of applying GC analysis, rooted in the theory of linear regression, to nonlinear neural systems. It has been theoretically established in previous works that GC, despite being a linear method, possesses the capability to extract nonlinear causal interactions within neural systems under certain conditions, and can be applied to reconstructions of the structural connectivity of neuronal networks [2, 8, 12, 33, 43].

However, it has been found that the GC value depends on the sampling interval of the time series, and it may even infer a false causal relationship when the sampling interval is not properly chosen [26]. Further mathematical analysis has been developed to reveal the dependence of GC on sampling interval for a pair of neurons, which showed that the bivariate GC converges to zero linearly as the sampling interval approaches zero. In addition, a practical approach was proposed to reliably estimate the causal relations between nodes in the network [44, 45]. It successfully overcomes the issue of vanishing GC caused by small sampling interval, i.e. high sampling rate in recent experimental measurements, when the original system satisfies certain conditions.

It is noted that the previous analysis mainly focused on the conventional GC on a bivariate system, which limits its applicability to large neural systems composed of more than two variables. In this work, we extend the mathematical analysis to the case of multivariate conditional GC, and show that the asymptotic structure of conditional GC with