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IMAGE SMOOTHING VIA A NOVEL ADAPTIVE WEIGHTED L_0 REGULARIZATION

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Abstract. Image smoothing has been extensively used in various fields, e.g., edge extraction, image abstraction, and image detail enhancement. Many existing optimization-based image smoothing methods have been proposed in recent years. The downside of these methods is that the results often have unclear edges and missing structures. To obtain satisfactory smoothing results, we design a novel optimization model by introducing an anisotropic L_0 gradient intensity. Specifically, a weighted matrix $\mathbf{T}$ is imposed to control further the sparsity of the gradient measured by L_0-norm. Since the proposed model is non-convex and non-smooth, we apply the half quadratic splitting (HQS) algorithm to solve it effectively. In addition, to obtain a more suitable regularization parameter λ, we utilize an adaptive parameter selection method based on Morozovs discrepancy principle. Finally, we conduct numerical experiments to illustrate the superiority of our method over some state-of-the-art methods.

Key words. Image smoothing, adaptive weighted matrix, L_0 gradient minimization, parameter selection.

1. Introduction

Image smoothing has a broad variety of applications as basic visual research, such as image detail enhancement [8], edge extraction [41], clip-art compression artifact removal [39, 32], image denoising [15, 31, 37], image segmentation [30, 33] and image abstraction [38]. Image smoothing aims to obtain an image with complete structural content but without small textures, which is undoubtedly challenging. Actually, many advanced image smoothing algorithms have been studied over recent years [4, 46, 22].

Model-based methods have been paid much attention because of their excellent performance and solid theoretical guarantee. The optimization model related to image smoothing is usually written as

$$\min_u \|u - f\|_2^2 + \lambda\varphi(u), \tag{1}$$

where f and u denote the input image and the resulting smoothed image, respectively. λ is a tradeoff parameter used to adjust the degree of regularization. Many methods are devoted to selecting an appropriate regularizer to smooth the image more effectively. Xu *et al.* [39] used L_0-norm to remove a globally small-magnitude gradient with edge preservation. However, this method failed to deal with small structures with large amplitude and small resolution. The relative total variation (RTV) [41] based regularization is employed to extract main structures under the complex texture. Liu *et al.* [20] combined the L_0 sparse constraint with the nonlocal constraint to remove the fine texture of the image. However, the above methods

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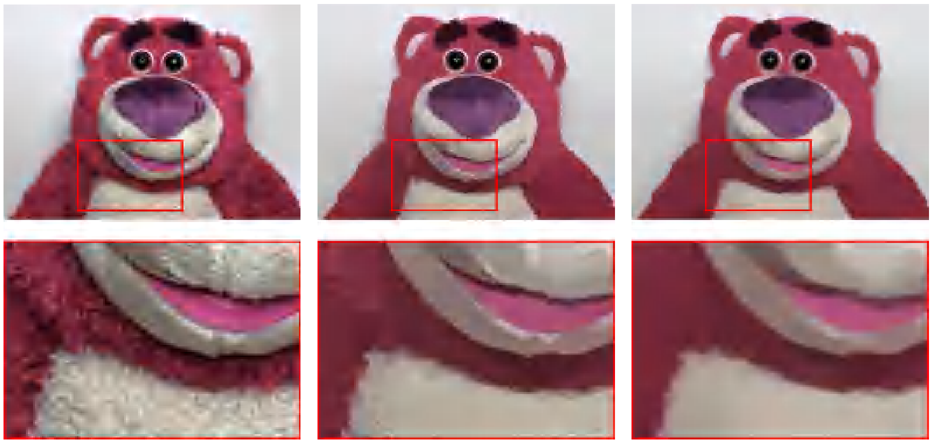
are not flexible enough in gradient processing, which hinders their ability to protect weak edges and eliminate fine textures effectively. In Figure 1(b), we show the limitation of the L_0 smoothing.



(a) Input (b) L_0 [39] (c) Ours

FIGURE 1. An example of L_0 smoothing [39] and our method. The result of [39] can not remove small structures. In contrast, our method achieves a more convincing result.

Total variation (TV) regularization is also capable of smoothing. However, it tends to lose local structure and produce artifacts. To cope with this barrier, many previous works that combine anisotropic filtering with TV regularization have been proposed [25, 43]. In [26], the authors employ the characteristics of anisotropic total variation (ATV) to remove noise while retaining the complete structure:

$$\min_u \|u - f\|_2^2 + \lambda \|\mathbf{T}\nabla u\|_1, \tag{2}$$

where $\mathbf{T}(x, y) := \text{diag}(t_1(x, y), t_2(x, y))$ and (x, y) denotes the pixel location of u. For each pixel, the gradient of the image $\nabla u = (\nabla_x u, \nabla_y u)$ is the difference of adjacent pixels along the x-axis and y-axis. In fact, $\mathbf{T}(x, y)$ can impose different penalty weights on $\nabla_x u(x, y)$ and $\nabla_y u(x, y)$. In this work, we try to introduce the weighted matrix $\mathbf{T}(x, y)$ into our smoothing model, i.e., we combine anisotropic filtering with L_0 smoothing. The proposed adaptive weighted L_0 regularization model is as follows:

$$\min_u \|u - f\|_2^2 + \lambda \left\|\mathbf{T}\nabla u\right\|_0, \tag{3}$$

where $\|\cdot\|_0$ denotes the L_0-norm and λ is the regularization parameter. This regularization term can flexibly handle gradients and depict local details of images, such as weak edges and small structures. As is well known, the parameter λ also has an impact on the image smoothing effect. A large λ can cause the image to be too smooth and weak edges to disappear, while a small λ cannot remove unrelated textures that exist in the image. A good optimization model should have an appropriate regularization parameter. Common parameter adjustment methods include the L-curve-based approach [18], the generalized cross-validation method [10], the variational Bayesian method [1], and Morozovs discrepancy principle [34, 24]. Specifically, Morozovs discrepancy principle is one of the most widely used