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DECOUPLING METHODS FOR FLUID-STRUCTURE INTERACTION WITH LOCAL TIME-STEPPING

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Abstract. We introduce two global-in-time domain decomposition methods, namely the Steklov-Poincare method and the Robin method, for solving a fluid-structure interaction system. These methods allow us to formulate the coupled system as a space-time interface problem and apply iterative algorithms directly to the evolutionary problem. Each time-dependent subdomain problem is solved independently, which enables the use of different time discretization schemes and time step sizes in the subsystems. This leads to an efficient way of simulating time-dependent phenomena. We present numerical tests for both non-physical and physical problems, with various mesh sizes and time step sizes to demonstrate the accuracy and efficiency of the proposed methods.

Key words. Fluid-structure interaction, domain decomposition, local time stepping.

1. Introduction

The Fluid-Structure Interaction (FSI) problems are multiphysics problems where the fluid flow and an elastic structure are coupled through the continuity of traction force and velocity on the interface. FSI systems have a wide range of applications in various fields, including manufacturing, energy, aeroelasticity, defense, and biology [5, 7, 11, 12, 18, 22, 29, 37, 40]. In engineering, FSI systems are considered in designing inkjet printers, blades for wind turbines, airplane wings, combustion chambers in engines, and offshore oil rigs. In biology, such systems are often considered to study blood flow through vessels.

The FSI system is considered as a coupled monolithic system in [3, 20, 27, 34, 35]. In such an approach, the computational complexity arises from solving a large matrix system, necessitating the use of an efficient and suitable preconditioner for the discretized system [35]. An alternative approach involves decoupling the fluid and structure subsystems [2, 4, 6, 8, 9, 10, 14, 15, 30, 32, 36]. Implementing such methods, despite their advantages of using partitioned solvers and smaller matrices for each subsystem, can pose challenges in achieving efficient iteration between the two subsystems.

There have been extensive studies on domain decomposition (DD) techniques for FSI in the literature. Various approaches have been considered, including explicit schemes [10, 14] and semi-implicit schemes [4, 15, 38]. Many implicit DD methods have also been investigated for better stability of the numerical solution. For example, an implicit DD method based on optimization is considered in [36], for both linear and nonlinear elastic formulations for the structure. There, the stress force on the interface is used as a Neumann control, which is updated until the stress discontinuity on the interface is sufficiently small, by enforcing the continuity of velocity through a Dirichlet boundary condition for the fluid subsystem. This process requires solving the subsystems in serial. Another optimization approach

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for FSI is explored in [30] by formulating the FSI problem as a least squares problem, where the jump in the velocities of the two substructures is minimized by a Neumann control enforcing the continuity of stress on the interface. In [39], the hybridizable discontinuous Galerkin (HDG) finite element method is used in the simulation of FSI. The coupling between an underlying incompressible fluid and an embedded solid is formulated through the overlapping domain decomposition method in conjunction with a mortar approach in [21]. A fictitious domain approach, where the fluid velocity and pressure are extended into the solid domain by introducing new unknowns, has been applied to study FSI in [2, 6, 32]. In [9] a splitting scheme based on Robin conditions is analyzed with an additional variable representing the structure velocity, where a common Robin parameter is utilized for both fluid and structure sub-problems. This approach uses common time steps for the fluid and the structure sub-problems, and the loosely coupled subproblems are solved at each time step. In [8], the finite element approximation of the DD formulation introduced in [9] is analyzed, and an error estimate is derived for the fully discretized system. Loosely coupled schemes based on interface conditions of Robin type are also found in [16] for the time-discretized FSI system, where the choice of optimal Robin parameters is analyzed.

In classical DD approaches for time-dependent problems, model equations are discretized uniformly in time, and DD methods are implemented at each time step as a steady-state problem. However, using a uniform time step throughout the entire time domain can be inefficient in certain FSI applications where the time scales of the fluid and structure domains differ. In recent studies, alternative approaches based on global-in-time or space-time domain decomposition (DD) have been employed where iterative algorithms are directly applied to the evolutionary problem. This enables the independent solving of subdomain problems with the different time discretization schemes and step sizes, resulting in an efficient simulation of time-dependent phenomena.

The space-time DD approach has been extensively investigated for porous medium flows (see [23, 24, 28] and the references therein) and recently studied for the Stokes-Darcy systems [25, 26]. In [26], a global-in-time DD method is developed based on the physical transmission conditions for the nonlinear Stokes-Darcy coupling. A time-dependent Steklov-Poincare type operator is constructed, and nonmatching time grids are implemented using L^2 projection functions to exchange data on the space-time interface between different time grids. Another global-in-time DD method is proposed in [25] for the mixed formulation of the non-stationary Stokes-Darcy system based on Robin transmission conditions.

This work aims to study the global-in-time DD methods introduced in [25, 26] for an FSI system using nonconforming time discretization. We consider two different DD schemes, based on the Steklov-Poincaré operator and Robin transmission conditions, respectively. To our knowledge, the global-in-time DD scheme has not been considered for an FSI system in the literature. Apart from the advantage of using local time stepping, another key point of this work is that we were able to simulate hemodynamic application problem using the Steklov-Poincaré method without encountering the stability issue. In the analysis of the DD method for the FSI system, additional difficulty arises due to the model equations of hyperbolic and parabolic types, unlike the Stokes-Darcy system. Another issue is caused by the lack of regularity of the unknown Lagrange multiplier function.

The paper is structured as follows. Section 2 introduces the FSI model system. In Section 3, we derive space-time interface problems for the continuous FSI model,