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NUMERICAL NS EQUATIONS-CONSTRAINED POWER OPTIMIZATION OF WIND TURBINES AFFECTED BY WAKE

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Abstract. We develop a model for wind farm power optimization while considering the wake interaction among wind turbines. The proposed model is a Navier-Stokes equations-constrained optimization model with the objective of maximizing the total power where the operating points of the turbines are the decision variables, and the three-dimensional vorticity-velocity Navier-Stokes equations of wind speed are among the constraints. Moreover, we develop an efficient numerical algorithm to solve the optimization model. This algorithm is based on the pattern search method, the actuator line method and a time-stepping scheme which is used to solve the vorticity-velocity Navier-Stokes equations. In the numerical experiments, we first compute the power generation of a commercial wind turbine called WindSpot for different wind speed. It is shown that the computed power is in a good agreement with the measurements. Then, in the case of two turbines, we find that by optimizing the turbines' operation while considering the wake effect, we can gain an additional 8.11% in the total power when the incoming wind speed on the boundary is 10 m/s.

Key words. Wind farm power optimization, the vorticity-velocity Navier-Stokes equation, wake interaction, time-stepping, pattern search, optimization algorithm.

1. Introduction

Currently, wind turbines are operating at their own local optimum points to maximize their own performance. Many studies have shown that operating all turbines in a wind farm at their local optimum points leads to the suboptimal performance of the overall wind farm [1, 2]. This is due to the wake generated by upstream wind turbines which alter the flow field and lead to a wind velocity deficit in downstream wind turbines [3–5]. As a consequence, if all wind turbines operate at their own local optimum points then the downstream wind turbines cannot generate power as much as the upstream wind turbines. For instance, Neustadter and Spera [7] investigated the performance of three turbines separated by seven rotor diameters. They found that if all turbines operate at their own local optimum points then the power loss of downstream turbines can be as high as 10%. Another investigation by Rebecca [8] shows that the power loss of downstream wind turbines in full wake conditions can be as high as 30%, but when averaged over different wind directions, it is around 5-8%. These studies confirm that operating turbines at their local optimum points will lead to suboptimal performance of the overall wind farm. Therefore, in order to improve the performance of the overall wind farm, it is necessary to find the global optimum points of wind turbines by optimizing the total power while taking into account the impact of the wake on power production. In this regard, Patricio [9] studied the power optimization while considering the wake impact where the two-dimensional Navier-Stokes equations were used to model the airflow around turbines, where the study employed an inefficient grid search method to find optimum operating points of the upstream wind turbine while assuming that the downstream turbine is operating at its own local optimum points for the two

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dimensional problem. There was no work on the realistic problems of the airflow around turbines in three dimensions modelled by three dimensional Navier-Stokes flow equations. In the framework of wind farm power optimization, in most of the studies, the total power is not explicitly optimized or the wake of wind turbines is modeled via an improvised BEM-alike method which has some limitations [7–11]. In these previous studies, there was few work to tackle the problem of explicitly optimizing the total power output in a wind farm. Thus, it is important to develop the wind power optimization where the total power will be explicitly optimized over multiple wind turbines in a wind farm.

In this paper, we develop a NS equations-constrained power optimization model over multiple wind turbines, for optimizing the total power output in a wind farm, by incorporating explicitly wake interactions among wind turbines. The important features are that the objective function of the total power is derived from the wakes of wind turbines by combining with the three dimensional vorticity-velocity Navier-Stokes equations as a constrain. The model optimizes on turbine operating while accounting for wake effects. It solves the optimization problem by embedding the three dimensional Navier-Stokes equations in the vorticity-velocity form with blade forces, which efficiently handles the wake effects in optimization. The proposed model is a PDE-constrained optimization model with the objective of maximizing the total power where the operating points of the turbines are the decision variables, and the constrained vorticity-velocity Navier-Stokes equations are used to compute the airflow as well as interacting wakes in the wind farm where the blade forces represent the loading of wind turbines. In the approach, a time-stepping finite difference scheme discretizes the three dimensional vorticity-velocity Navier-Stokes equations, where the false-transient technique ensures numerical stability, while the pattern-search derivative-free method is to solve the optimization procedure. The proposed algorithm of optimization approach is efficient and capable of handling the wind turbines in wind farms.

In numerical experiments, we first compute the power generation of a commercial wind turbine called WindSpot for different wind speed. It is shown that the computed power is in a good agreement with the measurements. We also show some numerical results that characterize the wake structure of the WindSpot. Furthermore, we find the global optimal operating points of multiple turbines operating in a wind farm. In the case of two turbines, we find that by optimizing the turbines' operation while considering the wake effect, we can gain an additional 8.11% in the total power when the incoming wind speed on the boundary is $10m/s$.

The paper is organized as the follows. In Section 2, we present the problem and the optimization model. In Section 3, we describe how to derive the aerodynamic forces using tabulated airfoil data. In Section 4, we present our numerical approach for solving the three-dimensional Navier-Stokes equations in the vorticity-velocity form. In Section 5, we describe the joint optimization algorithm for maximizing the total power production. In Section 6, we present the numerical case studies. Finally, conclusions are addressed in Section 7.