

Multisection Technique to Solve Interval-valued Purchasing Inventory Models without Shortages

Susovan Chakrabortty¹, Madhumangal Pal², Prasun Kumar Nayak³

^{1,2}Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore-721 102, India

³ Bankura Christian College, Bankura, 722 101, India,

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Abstract. This paper investigates an interval valued economic order quantity (EOQ) problem without shortage. Since it is almost impossible to find an analytic method to solve the proposed model, an optimization algorithm is designed. First, a brief survey of the existing works on comparing and ranking any two interval numbers on the real line is presented. Finally, the effectiveness of the designed algorithm is illustrated by a numerical example.

Key words: Inventory, Interval Number, Demand, Production, Simulation

1. Introduction

The economic order quantity (EOQ) model is first introduced by F.Harris [4]. Inventory control is an important field in supply chain management, since it can help companies reach the goal of ensuring delivery, avoiding shortages, helping sales at competitive prices and so forth. A proper control of inventory can significantly enhance a company's profit. To control an inventory system, one cannot ignore demand monitoring since inventory is partially driven by demand, and as suggested by Lau and Lau [2] in many cases a small change in the demand pattern may result in a large change in optimal inventory decisions. A manager of a company has to investigate the factors that influence demand pattern, because customers' purchasing behavior may be affected by factors such as selling price, inventory level, seasonality, and so on.

A large number of academic papers (for a review, see [11]) have been published describing numerous variations of the basic EOQ model. The body of the research assumes that the parameters involved in the EOQ model, such as the demand and the purchasing cost, are crisp values or random variables. However, in reality, the demand and the cost of the items often change slightly from one cycle to another.

For example, inventory carrying cost may be different in rainy season compared to summer or winter seasons (costs of taking proper action to prevent deteriorations of items in different seasons and also the labour charges in different seasons are different). Ordering cost, being dependent on the transportation facilities may also vary from season to season. Changes in the price of fuels, mailing charges, telephonic charges may also make the ordering cost fluctuating. Unit purchase cost is highly dependent on the costs of raw materials and labour charges, which may fluctuate over time.

To solve the problem with such imprecise numbers, stochastic, fuzzy and fuzzy-stochastic approaches [5, 6, 9, 10] may be used. In stochastic approach, the parameters are assumed to be random variables with known probability distribution. In fuzzy approach, the parameters, constraints and goals are considered as fuzzy sets with known membership functions. On the other hand, in fuzzy-stochastic approach, some parameters are viewed as fuzzy sets and others, as random variables. However, if the membership function of the fuzzy variable is complex, for example when a trapezoidal fuzzy number and a Gaussian fuzzy number coexist in a model, it is hard to obtain the membership function of the total cost. Therefore, these membership functions play a significant role in the method. However, in practice one may not be able to get exact membership function for fuzzy values and probability distribution for stochastic variable. Since precise

¹ Corresponding author. E-mail address: susovan_chakrabortty@ymail.com,

² E-mail address: mmpalvu@gmail.com

³ E-mail address: nayak_prasun@rediffmail.com

information is required, the lack of accuracy will affect the quality of the solution obtained. For these reasons, we have represented the imprecise number by interval numbers [3, 13].

Thus, the interval number theory, rather than the traditional probability theory and fuzzy set theory, is well suited to the inventory problem. According to the decision maker's point of view under changeable conditions, we may replace the real numbers by the interval valued numbers to formulate the problems more appropriately.

We organize the paper as follows : In section 2, we give some basic definitions, notations and comparison on interval numbers. In section 3, we give the model formulation and the solution procedure.

2. Interval number

An interval number proposed by Moore [13], is considered as an extension of a real number and as a real subset of the real line $\mathfrak{R}$.

Definition 1. An interval number $\tilde{A}$ is a closed interval defined by $\tilde{A} = [a_L, a_R] = \{x \in \mathfrak{R} : a_L \leq x \leq a_R; \mathfrak{R} \text{ be the set of all real numbers}\}$. The numbers a_L, a_R are called respectively the lower and upper limits of the interval $\tilde{A}$. An interval number A alternatively represented in mean-width or center-radius form as

$$\tilde{A} = \langle m(\tilde{A}), w(\tilde{A}) \rangle = \{x \in \mathfrak{R} : m(\tilde{A}) - w(\tilde{A}) \leq x \leq m(\tilde{A}) + w(\tilde{A})\}, \quad (1)$$

where $m(\tilde{A}) = \frac{1}{2}(a_L + a_R)$ and $w(\tilde{A}) = \frac{1}{2}(a_R - a_L)$ are the mid-point and half-width of the interval $\tilde{A}$.

Actually, each real number can be regarded as an interval, such as, for all $x \in \mathfrak{R}$, x can be written as an interval $[x, x]$, which has zero length.

The set of all interval numbers in $\mathfrak{R}$ is denoted by $I(\mathfrak{R})$.

2.1. Basic interval arithmetic

Let $\tilde{A} = [a_L, a_R] = \langle m_1, w_1 \rangle$ and $\tilde{B} = [b_L, b_R] = \langle m_2, w_2 \rangle \in I(\mathfrak{R})$ be two interval numbers, then

$$\tilde{A} + \tilde{B} = [a_L + b_L, a_R + b_R]; \quad \tilde{A} + \tilde{B} = \langle m_1 + m_2, w_1 + w_2 \rangle.$$

The multiplication of an interval by a real number $c \neq 0$ is defined as

$$c\tilde{A} = [ca_L, ca_R]; \quad \text{if } c \geq 0 \text{ and } c\tilde{A} = [ca_R, ca_L]; \quad \text{if } c < 0.$$

$$c\tilde{A} = c\langle m_1, w_1 \rangle = \langle cm_1, |c|w_1 \rangle.$$

The difference of these two interval numbers is

$$\tilde{A} - \tilde{B} = [a_L - b_R, a_R - b_L].$$

The product of these two distinct interval numbers is given by

$$\tilde{A}\tilde{B} = [\min\{a_L b_L, a_L b_R, a_R b_L, a_R b_R\}, \max\{a_L b_L, a_L b_R, a_R b_L, a_R b_R\}].$$

The division of these two interval numbers with $0 \notin \tilde{B}$ is given by

$$\tilde{A} \setminus \tilde{B} = \left[\min\left\{\frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R}\right\}, \max\left\{\frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R}\right\} \right].$$

2.2. Comparison between interval numbers

Let $\tilde{A} = [a_L, a_R] = \langle m_1, w_1 \rangle, \tilde{B} = [b_L, b_R] = \langle m_2, w_2 \rangle$ be two interval numbers within $I(\mathfrak{R})$. These two intervals may be one of the following types:

1. Two intervals are completely disjoint (non-overlapping).
2. Two intervals are nested, (fully overlapping).
3. Intervals are partially overlapping.

A brief comparison on different interval orders is given in [1, 12].