

Intuitionistic Neutrosophic Set Relations and Some of Its Properties

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Abstract. In this paper, we define intuitionistic neutrosophic set (INSs). In fact, all INSs are neutrosophic set but all neutrosophic sets are not INSs. We have shown by means of example that the definition for neutrosophic sets the complement and union are not true for INSs also give new definition of complement, union and intersection of INSs. We define the relation of INSs and four special type of INSs relations. Finally we have studied some properties of INSs relations.

Keywords: Neutrosophic sets, intuitionistic fuzzy sets, intuitionistic neutrosophic sets, intuitionistic neutrosophic relations

1. Introduction

In 1965 [7], Zadeh first introduced the concept of fuzzy sets. In many real applications to handle uncertainty, fuzzy set is very much useful and in this one real value $\mu_A(x) \in [0,1]$ is used to represent the grade of membership of a fuzzy set A defined on the universe of discourse X . After two decades Turksen [13] proposed the concept of interval-valued fuzzy set. But for some applications it is not enough to satisfy to consider only the membership-function supported by the evident but also have to consider the non-membership-function against by the evident. Atanassov [3] introduced another type of fuzzy sets that is called intuitionistic fuzzy set (IFS) which is more practical in real life situations. Intuitionistic fuzzy sets handle incomplete information i.e., the grade of membership function and non-membership function but not the indeterminate information and inconsistent information which exists obviously in belief system.

Wang et.al. [2] introduced another concept of imprecise data called neutrosophic sets. Neutrosophic set is a part of neutrosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic set is a powerful general formal framework that has been recently proposed. In neutrosophic set, indeterminacy by the evident is quantified explicitly and in this concept membership, indeterminacy membership and non-membership functional values are independent. Where membership, indeterminacy membership and non-membership functional values are real standard or non-standard subsets of $]^-0,1^+]$.

In real life problem which is very much useful. For example, when we ask the opinion of an expert about certain statement, he or she may assign that the possibility that the statement true is 0.5 and the statement false is 0.6 and he or she not sure is 0.2. This idea is very much needful in a various problem in real life situation.

The neutrosophic set generalized the concept of classical set, fuzzy set [7], interval-valued-fuzzy set [13], intuitionistic fuzzy set [3], etc. Recently Bhowmik and Pal et.al. [14] have defined intuitionistic neutrosophic set.

Definition 1 Let X be a fixed set. A FS A of X is an object having the form $A = \{\langle x, \mu_A(x) \rangle / x \in X\}$. where the function $\mu_A : X \rightarrow [0,1]$ define the degree of membership of the element $x \in X$ to the set A ,

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which is a subset of X .

Definition 2 Let X be a fixed set. An IFS A of X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$, where the function $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define respectively the degree of membership and degree of nonmembership of the element $x \in X$ to the set A , which is a subset of X and for every $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

An element x of X is called significant with respect to a fuzzy subset A of X if the degree of membership $\mu_A(x) > 0.5$, otherwise, it is insignificant. We see that for a fuzzy subset A both the degrees of membership $\mu_A(x)$ and non-membership $\nu_A(x) = 1 - \mu_A(x)$ can not be significant. Further, for an IFS $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ it is observe that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, for all $x \in X$ and hence it is observed that $\min\{\mu_A(x), \nu_A(x)\} \leq 0.5$, for all $x \in X$.

Definition 3 [12] Let X be a fixed set. A generalized intuitionistic fuzzy set (GIFS) A of X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ where the function $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define respectively the degree of membership and degree of nonmembership of the element $x \in X$ to the set A , which is a subset of X and for every $x \in X$ satisfy the condition

$$\mu_A(x) \wedge \nu_A(x) \leq 0.5, \text{ for all } x \in X.$$

This condition is called generalized intuitionistic condition (GIC). In fact, all GIFs are IFSs but all IFSs are not GIFs.

Having motivated from this definition we propose another concept of neutrosophic set.

In this paper, in Section 2 we recall the non-standard analysis by Abraham Robinson and some definitions of neutrosophic sets of Wang et.al. [2]. In Section 3, we define a new type of neutrosophic sets called intuitionistic neutrosophic sets (INSs) and have shown by means of example, that the definition for neutrosophic sets the complement and union are not true for INSs. Also we define new definition of complement, union and intersection of INS. In section 4, we define the relation of INSs and four special type of INSs relations. Finally we have studied some properties of INSs relations.

2. Preliminaries

In 1960s Abraham Robinson has developed the non-standard analysis, a formalization and a branch of mathematical logic, that rigorously defines the infinitesimals. Informally, an infinitesimal is an infinitely small number. Formally, x is said to be infinitesimal if and only if for all positive integers n one has $|x| < \frac{1}{n}$. Let $\varepsilon > 0$ be a such infinitesimal number. The hyper-real number set is an extension of the real number set, which includes classes of infinite numbers and classes of infinitesimal numbers. Let's consider the non-standard finite numbers $1^+ = 1 + \varepsilon$, where "1" is its standard part and " ε " is non-standard part and $^-0 = 0 - \varepsilon$, where "0" is its standard part and " ε " is non-standard part. Then we call $]^-0, 1^+[$ is a non-standard unit interval.

Generally, the left and right boundaries of a non-standard interval $]^-a, b^+[$ are vague and imprecise. Combining the lower and upper infinitesimal non-standard variable of an element we can define as $^-c^+ = \{(c - \varepsilon) \cup (c + \varepsilon)\}$.

Addition of two non-standard finite numbers with themselves or with real numbers defines as:

$$^-a + b = ^-(a + b), a + b^+ = (a + b)^+, ^-a + b^+ = ^-(a + b)^+, ^-a + ^-b = ^-(a + b), a^+ + b^+ = (a + b)^+.$$

Similar for subtraction, multiplication, division, root and power of non-standard finite numbers with themselves or real numbers.

Now we recall some definitions of Wang et al.[2].

Let X be a space of points (objects), with a generic elements in X denoted by x . Every element of X is characterized by a truth-membership function T , an indeterminacy function I and a falsity-membership function F . Where T, I and F are real standard or non--standard subsets of $]^-0, 1^+[$, that is,