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A Numerical Algorithm for Solving an Inverse Nonlinear Parabolic Problem

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Abstract. In this paper, we propose an algorithm for numerical solving an inverse nonlinear diffusion problem. The algorithm is based on the Laplace transform technique and the finite difference method in conjunction with the least-squares scheme. To regularize the resultant ill-conditioned linear system of equations, we apply the Tikhonov regularization method to obtain the stable numerical approximation to the solution. To show the efficiency and accuracy of the present method a test problem will be studied.

Keywords: Inverse nonlinear parabolic problem, Laplace transform, Finite difference method, Least-squares method, Regularization method.

1. Introduction

Inverse heat conduction problems (IHCPs) appear in many important scientific and technological fields. Hence analysis, design implementation and testing of inverse algorithms are also are great scientific and technological interest. Mathematically, the inverse problems belong to the class of problems called the ill-posed problems. That is, their solution does not satisfy the general requirement of existence, uniqueness, and stability under small changes to the input data. To overcome such difficulties, a variety of techniques for solving IHCPs have been proposed.

Numerical solution of an inverse nonlinear diffusion problem requires to determine an unknown diffusion coefficient from an additional information. These new data are usually given by adding small random errors to the exact values from the solution to the direct problem. This paper presents the inverse determination of the diffusion coefficient of an unknown porous medium[1].

Mathematically, IHCPs belong the class of ill-posed problems, i.e. small errors in the measured data can lead to large deviations in the estimated quantities. The physical reason for the ill-posedness of the estimation problem is that variations in the surface conditions of the solid body are damped towards the interior because of the diffusive nature of heat conduction. As a consequence, large-amplitude changes at the surface have to be inferred from small-amplitude changes in the measurements data. Errors and noise in the data can therefore be mistaken as significant variations of the surface state by the estimation procedure. Therefore the IHCP has a unique solution, but this solution is unstable. In this paper this instability is overcome using the Tikhonov regularization method with L-curve criterion for the choice of the regularization parameter.

The outline of this paper is as follows. In the section 2, we formulate an inverse nonlinear parabolic problem. In the section 3, we linearize nonlinear term by Taylor's series expansion, remove time-dependent terms by Laplace transform technique, discretize governing equations by finite difference method and used least squares method for correction unknown coefficients. Numerical experiments in section 4, confirm our theoretical results for an unknown porous medium.

2. Mathematical model

The mathematical model of an inverse nonlinear parabolic problem with initial and boundary conditions is the following form

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$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(a(u)\frac{\partial u}{\partial x}\right), 0<x<1, 0<t<T, \tag{1}$$

$$u(x,0)=p(x), 0<x<1, \tag{2}$$

$$\frac{\partial u(0,t)}{\partial x}=g(t),\ 0<t<T, \tag{3}$$

$$\frac{\partial u(1,t)}{\partial x}=q(t),\ 0<t<T, \tag{4}$$

$$u(1,t)=f(t),\ 0<t<T, \tag{5}$$

where T is a given positive constant, and $g(t)$, $p(x)$ and $q(t)$are piecewise-continuous known functions, while $u(x,t)$ and diffusion coefficient $a(u(x,t))>0$, [2], are unknown which remain to be determined.

For an unknown function $a(u)$ we must therefore provide additional information (5) to provide a unique solution $(u,a(u))$ to the inverse problem (1)-(5). Parabolic problems including equation (1) have been previously treated by many authors who considered certain special case of this type of problem [6-11]. In this article, under certain conditions on $g(t)$, $p(x)$, $q(t)$ and $f(t)$, we shall identify both $u(x,t)$ and diffusion coefficient $\mathrm{a}(\mathrm{u})$ at any time by using the over specified condition (5), initial and boundary conditions (2)-(4).

3. Description of the numerical scheme

Consider the one-dimensional nonlinear problem described by the problem (1)-(5), where (1) is nonlinear. The application of the present numerical method to find the solution of problem (1)-(5), can be divided into the following steps.

3.1. Linearizing the nonlinear terms

Since the application of the Laplace transform technique is only restricted to the linear system, so that the nonlinear term in equation (1) must be linearized. Therefore, we used Taylor's series expansion for linearized nonlinear terms and we obtain [9]

$$\frac{\partial}{\partial x}\left(a(u)\frac{\partial u}{\partial x}\right)=\frac{\partial}{\partial u}\left(k(u)\right)_{u=\bar{u}}\frac{\partial^2 u}{\partial x^2}=a(\bar{u})\frac{\partial^2 u}{\partial x^2}, \tag{6}$$

where

$$k(u)=\int_0^u a(\rho)d\rho, \tag{7}$$

is a nonlinear function and $\bar{u}=(\bar{u}_0,\bar{u}_1,\cdots,\bar{u}_N)$ denotes the previously iterated solution.

3.2. Remove time dependent terms

For remove time dependent terms from equations (2) , (3) , (4) and (6) the method of the Laplace transform is employed. Therefore we obtain

$$a(\bar{u})\frac{\partial^2\tilde{u}}{\partial x^2}=s\tilde{u}-p(x), 0<x<1, \tag{8}$$

$$\frac{\partial\tilde{u}}{\partial x}=G(s),\ x=0, \tag{9}$$

$$\frac{\partial\tilde{u}}{\partial x}=Q(s),\ x=1, \tag{10}$$