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A New Skew Linear Interpolation Characteristic Difference Method for Sobolev Equation

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Abstract. A new kind of characteristic-difference scheme for Sobolev equations is constructed by combining characteristic method with the finite-difference method and with the skew linear interpolation method. The convergence of the characteristic-difference scheme is studied. The advantage of this scheme is very effectual to eliminate the numerical oscillations and have potential advantages in other equations.

Keywords: Sobolev equation; characteristic-difference scheme; skew linear interpolation; convergence

1. Introduction

Many mathematical physics problems can be described by Sobolev equations, such as in fluid flow, heat diffusion and other areas of application. The primal numerical solution of using finite difference method and finite element method for one dimensional Sobolev equations is in [1],[2]. In the year of 1982, Douglas and Russel [3] presented the method of characteristics with finite element or finite difference procedures to solve convection diffusion equations, and then You [4] applied this characteristics difference element method to solve Sobolev equation. During the computation, this method used the algebraic interpolation of the last time step, thus for some problems the stability of the computational scheme is not good enough, even can cause some numerical oscillation. To avoid happen the phenomena of numerical oscillation, Qin [5] introduced a new linear interpolation method (see figure 1) in solving convection diffusion problem: use the values of four points $(x_{j-1},t_{n-1}),(x_j,t_{n-1}),(x_{j-1},t_n),(x_j,t_n)$ to make bilinear interpolation for the value of point P_*.

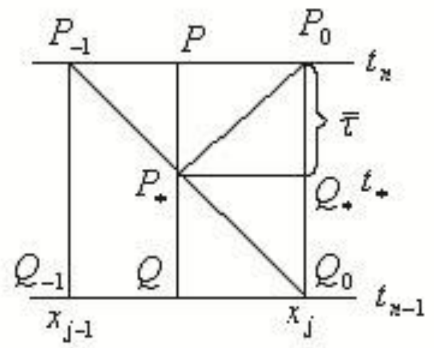

Figure 1. Bilinear interpolation

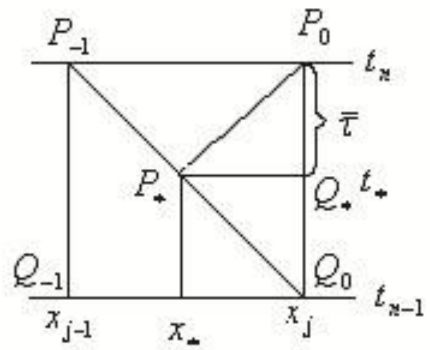

Figure 2. Skew linear inpolation

In this paper, we present a improved characteristic-difference method of [5](see figure 2) in solving the Sobolev equations: only use the values of two points $(x_j,t_{n-1}),(x_{j-1},t_n)$ (see figure 2) to make Skew linear interpolation of point P_*. Compare with the method in [3],[4], our new method show itself more stability. Compare with [5], our method is easier to realize in algorithm. This method can also be applied to solve

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convection dominated diffusion equations.

2. Construction of the finite difference scheme

Consider one dimensional initial-boundary Sobolev equations:

$$\begin{cases} c(x)\dfrac{\partial u}{\partial t}+b(x)\dfrac{\partial u}{\partial x}-\dfrac{\partial}{\partial x}\left(a(x)\dfrac{\partial u}{\partial x}+d(x)\dfrac{\partial^2 u}{\partial x\partial t}\right)=f(x,t),(x,t)\in(0,L)\times(0,T] \\ u(x,0)=u_0(x),x\in(0,L) \\ u(0,t)=g_0(t),u(L,t)=g_1,t\in[0,T] \end{cases} \tag{1}$$

where $a(x),b(x),c(x),d(x)\in C[0,L]$, $f(x,t)\in L^2([0,L]\times[0,T])$, and there exists positive constants a_0,b^*,c_0,d_0, $a(x)\ge a_0>0$, $|b(x)|\le b^*$, $c(x)\ge c_0>0$, $d(x)\ge d_0\ge 0$, for $\forall x\in[0,L]$.

The solvability and uniqueness of (1) can be found in [2]. We assume that (1) has a unique solution and have some necessary smoothness. Denote the characteristic direction of operator $c(x)\dfrac{\partial u}{\partial t}+b(x)\dfrac{\partial u}{\partial x}$ to be $\lambda=\lambda(x)$, and then the characteristic derivative is defined by

$$\frac{\partial}{\partial\lambda}=\frac{1}{\varphi(x)}\left[c(x)\frac{\partial}{\partial t}+b(x)\frac{\partial}{\partial x}\right],$$

where $\varphi(x)=\left[b(x)^2+c(x)^2\right]^{1/2}$. Therefore, the first equation of (1) can then be write as the following form:

$$\varphi(x)\frac{\partial u}{\partial\lambda}-\frac{\partial}{\partial x}\left[a(x)\frac{\partial u}{\partial x}+d(x)\frac{\partial^2 u}{\partial x\partial t}\right]=f(x,t),\quad 0<x<L,0<t<T. \tag{2}$$

In figure 2, suppose the values on $n-1$ time step is either initial value or already be computed by initial value. When $b(x)>0$, the characteristic direction at point P_0 is the direction along P_0P_*, where P_* is the intersection point of Q_0P_{-1} and characteristic direction. Thus by using linear interpolation, we can use the values at points Q_0 and P_{-1} to get the value at point P_*, and then applying finite difference method and characteristic method to construct an implicit difference scheme. The advantage of this method is: we only need to make skew linear interpolation in the segment Q_0P_{-1}, need not do other extra work to determine the interpolation point. This technique is better than the normal finite difference method by decreasing the truncation error along time.

Take space step $h>0$, mesh grid $x_j=jh$, $j=0,1,2,\cdots M=[L/h]$, time step $\tau>0$, mesh grid $t_n=n\tau$, $n=0,1,\cdots,\mathrm{N}=[\mathrm{T}/\tau]$, along the characteristic line P_0P_*, we make following finite difference discrete:

$$\left(\varphi\frac{\partial u}{\partial\lambda}\right)_j^n\approx\varphi_j\frac{u(x_j,t_n)-u(x_*,t_*)}{|P_0P_*|}, \tag{3}$$

where (x_*,t_*) denotes the coordinate at point P_*, $x_*=x_j-b_j\bar{\tau}/c_j$, $\varphi_j=\varphi(x_j)$. Denote $c_j=c(x_j)$, $b_j=b(x_j)$, $a_j=a(x_j)$, $d=d(x_j)$, $j=1,2,\cdots,M$. Thus from $\dfrac{|Q_0Q_*|}{|Q_0P_0|}=\dfrac{|P_*Q_*|}{|P_0P_{-1}|}$, we have $\bar{\tau}=\dfrac{c_jh\tau}{c_jh+b_j\tau}$, $t_*=n\tau-\bar{\tau}$. Therefore from

$$|P_0P_*|=\sqrt{(x_j-x_*)^2+\bar{\tau}^2}=\varphi_j\bar{\tau}/c_j,$$