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A Discontinuous Galerkin Level Set Method for Compressible two Fluid Flow

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Abstract. It is a crucial problem in the studies of the fluid on numerical simulations of the interfaces movement in multi-component fluids. In the present paper, a discontinuous Galerkin method is developed to simulate two-fluid flow. A level set method is used to capturing moving interfaces and a ghost fluid method with isobaric fix is used to disposing the interfaces boundary. Several test problems of two fluid flowing is solved and the comparisons between the numerical results and exact case are performed, which indicate the effectiveness of the method.

Key words: Multi-media flow; DG method; Level Set Method; GFM

1. Introductions

Discontinuous Galerkin method was initially introduced by Reed and Hill[1] to solve linear neutron transport equations. Lesaint and Raviart[2] were the first to put this method on a firm mathematical basis. Since that time, rigorous analyses of the method are made by Johnson[3], Richter[4], and by Peterson[5], Especially Cockburn and Shu introduced an explicit, nonlinearly stable high order Runge–Kutta type time discretization[6], which makes it to be an attractive method in computational fluid dynamics(CFD)[7-8].

But as we all know, when a well established numerical method for single-medium flow is applied directly to the multi-medium flow, there can arise severe nonphysical oscillations in the vicinity of the material interface especially in the presence of shock and large density ratio[9]. As such, numerical simulation of multi-medium flow is an important interesting area in recently science computation. Especially two-fluid, where two non-mixing fluids are separated by a sharp fluid interface, find many applications in both engineering and physics[10]. The main difficulty in computing multi-medium flow is how to treat the moving medium interface. A general classification of interface methods in multi-material flows simulation divides them into Lagrangian and Eulerian[10-12]

Lagrange methods[11] use the interface particles to characterize the interface and move with the fluid in order to capture the interface movement. It has extreme precision on the interface simulation. This method can accurately track the interface evolution and can discontinuously treat no numerical dissipation, but it is hard to treat the variation of interface topological structure and complicated to develop to high dimension. Euler methods[12] are effective to treat most problems and can treat the large deformation movement, but it is unable to precisely locate the multi-component interface and is unsuitable to the numerical simulation that requires high interface position. The robust ideal method must have both their merits.

A technique coupling level set method[13] and ghost fluid method(GFM) developed[12] by Osher, Sethian and Fedkiw provides an attractive alternative for multi-medial flow simulation. Level set method is used to capture the interface position, and GFM is used to define the state variables of neighboring mesh points on the interface. At the same time, an interface entropy interpolation technique is used to capture the suitable interface boundary conditions in order to decrease possible non-physical oscillation.

In the present paper, a discontinuous Galerkin(DG) method[6-8] is developed to simulate two fluid flow. In computational, a level set method is used to capturing moving interfaces and a GFM with isobaric fix is used to disposing the interfaces boundary. Several test problems of two fluid flowing is solved and the

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comparisons between the numerical results and exact case are performed, which indicate the effectiveness of the method.

The paper is organized as follows. We begin by describing the mathematical model in section 2 and the numerical discretisation of the Euler equations and level set function are presented in section 3, and then GFM with isobaric fix are given in section 4. Section 5 contains the associated numerical results. Finally, we sum up our conclusions in Section 6.

2. Mathematical model

2.1. governing equations

The basic equations for two-dimensional compressible flow are the 2D Euler equations,

$$\mathbf{U}_t + \nabla \cdot \mathbf{F} = 0, \qquad \Omega \times (0,T) \tag{1}$$

where $\mathbf{U} = [\rho, \rho u, \rho v, E]^{\mathrm{T}}$, $\mathbf{F}_x = [\rho u, \rho u^2 + p, \rho uv, (E+p)u]^{\mathrm{T}}$, $\mathbf{F}_y = [\rho v, \rho uv, \rho v^2 + p, (E+p)v]^{\mathrm{T}}$, $\Omega \in R^2$, T is time variable, ρ is density, u and v are velocity component of x and y direction respectively, E is the total energy per unit volume, p is the pressure, and E is the total energy per unit volume

$$E = p/(\gamma - 1) + \rho(u^2 + v^2)/2 \tag{2}$$

where γ is the ratio of specific heat.

2.2. Level Set equation

We use the level set equation

$$\frac{\partial \varphi}{\partial t} + u\frac{\partial \varphi}{\partial x} + v\frac{\partial \varphi}{\partial y} = 0 \tag{3}$$

to keep track of the interface location as the zero level of φ. And φ is usually chosen as a sign distance function defined as followed

$$\varphi(x,y,0) = \begin{cases} >0 & (x,y) \in \text{inside fluid } 1 \\ =0 & (x,y) \in \text{interface} \\ <0 & (x,y) \in \text{outside fluid } 1 \end{cases} \tag{4}$$

However, during its evolution, the level set function can lose the property of being the distance function[14]. So a re-initialized equation (5) is required to keep the function φ holding the characteristic of distance.

$$\frac{\partial \varphi(x,y,\tau)}{\partial \tau} + S(\varphi_0)(|\nabla \varphi| - 1) = 0 \tag{5}$$

where τ is fictitious time, $S(\varphi_0) = \varphi \big/ \sqrt{\varphi^2 + \theta^2}$ is a sign function and $\theta = \min(\mathrm{d}x, \mathrm{d}y)$.

The Equation (3) is numerically solved using the third-order RK-TVD scheme for time discretization and the fifth-order WENO scheme[15] for space discretization. Re-initialized equation (5) is solved using fifth-order WENO scheme developed by Peng et al[15].

3. The implementation of the DG method

First, the equations (1) are discretized in space by using a DG method. $\forall t \in [0,T]$, the approximate solution $\mathbf{U}_h(\boldsymbol{X}, t)$ is sought in the finite element space of discontinuous functions

$$V_h = \left\{ v_h \in L^\infty(\Omega) : v_h|_K \in V(K), \forall K \in \Gamma_h \right\}$$

where Γ_h is a finite partition of the domain Ω, $V(K)$ is the so-called local space, which is taken as the collection of polynomials of degree k, for $k = 2$, in this paper and $\boldsymbol{X} = (x, y)$.