

On General Binary Relation Based Rough Set

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Abstract. To keep the key idea of rough set and the representation of information in rough set theory, empty representation is processed properly and three new forms of rough approximation sets are defined as a generation of general binary relation based rough set model. Moreover, the properties of approximation operators in these new rough sets are discussed. The relations among them are studied in this paper. In addition, examples are arranged to interpret what we studied in this paper.

Keywords: General binary relation, Approximation operator, Rough set, Empty description, Duality

1. Introduction

Rough set theory was first proposed by Pawlak Z in 1980's. It is a mathematical tool to process information with uncertainty and vagueness. And it is also a useful soft computing tool in intelligence computing. The rough set theory can yet be regarded as a kind of more effective method to handle complex systems in data mining (DM) and knowledge discovery in database (KDD) [1,3,4,5]. Rough set theory, probability theory, fuzzy set theory and evidence theory are all tools to deal with uncertainty. Compared with other theories, the most significant difference is that no more prior information is needed but the specified information system for problems in rough set theory. Much better affections may come about in practical problems by combining rough set and other methods. Noise is a factor which can't be avoided in practice. Influenced by noise in data and limited by the requests of practical problems, the original rough set proposed by Pawlak is confined in practical applications. Many generalized rough set models have been proposed and studied systematically. The popularizations such as variable precision rough set, dominance-based rough set, tolerance-based rough set, fuzzy rough set, rough set based on covering, rough set based on evidence theory, probabilistic rough set, etc.[1,2,8,9,10,11,12,14], makes that rough set theory affects in more areas and fields. More useful information is being discovered and more values are being produced in real world. As researches on rough set are expanding in depth and further, the rough set theory is now being more and more abundant, theorized and systematic. Successful applications have been applied in many areas and fields, such as in subjects medical science, chemistry, materials science, geographical science, management, finance, conflicts resolutions, and so on. Excellent effects have been succeeded in many areas. Requirements in applications and the propelling of achievements are promoting rough set theory to be one of the most active research areas in information science and several interdisciplines [12,14,15].

Studying on general binary relation based rough set can make rough set theory more adaptable for generalized relations and produce more values in practice. Theories on general binary relation based rough set, which is denoted by GBRS, have been placed to some extent [6,7,14]. On the basis of Pawlak rough set, equivalence relations are popularized to general binary relations. And generalizations of GBRS are studied by constructive method, axiomatic method and the key idea of rough approximating in this paper. Properties of the corresponding approximation operators are discussed and proved. Moreover, examples are employed to help understand what we study in this paper.

2. Pawlak Rough Set

The classical rough set proposed by Pawlak is on the basis of equivalence relation on universe [3,4,5].

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Objects carry some unique information under relations and they can be classified by equivalence relations. Objects classified as the same class carry the information which is totally the same. If a concept can be presented by the union of some classes, that is the concept equals to the combination of some classes, then the concept carry all the information represented by the objects in these classes. Else, if a concept can't equal to the union of some classes, a pair of sets, which are constructed by the union of some classes, are employed to represent the concept approximately. One of the two sets is consisted of objects with totally confirm information that the concept carries. The selection of objects in this form relies on the precisely inclusion of classes to the concept. The other set is consisted of objects with information that possibly support the concept. The selection of these objects considers the nonempty joint between classes and the concept. The pair of sets is called, respectively, lower approximation set and upper approximation set [3,4,5,9,11,12,13]. Some basic definitions in Pawlak rough set will be illustrated for use in this paper.

Definition 2.1([3,14]) Let U be a nonempty finite set consisted of objects and called universe. For any $x \in U$, x is called an object. $R \subseteq U \times U$ is a binary relation on universe. x has the relation R with y if and only if $(x, y) \in R$, that is $xRy \Leftrightarrow (x, y) \in R$. If R satisfies

- (1) Reflective: $\forall x \in U, xRx$;
- (2) Symmetric: $xRy \Rightarrow yRx$;
- (3) Transitive: $xRy, yRz \Rightarrow xRz$;

then R is an equivalence relation on the universe.

Definition 2.2([3,14]) Let U be the universe. $R \subseteq U \times U$ is an equivalence relation on the universe. Then (U, R) is called Pawlak approximation space. For any $X \subseteq U$, X is called a concept on the universe. For any $x \in U$, $[x]_R = \{y \in U | (x, y) \in R\}$ is called the equivalence class of x with respect to R . $U/R = \{[x]_R | x \in U\}$ is called the partition induced by R to U .

Definition 2.3([4,14]) Let U be the universe. $R \subseteq U \times U$ is an equivalence relation on the universe. For any $X \subseteq U$, the lower approximation and upper approximation of X with respect to Pawlak approximation space (U, R) are defined, respectively, as

$$\begin{aligned}\underline{R}(X) &= \{x \in U | [x]_R \subseteq X\}, \\ \overline{R}(X) &= \{x \in U | [x]_R \cap X \neq \emptyset\}.\end{aligned}$$

X is definable with respect to R if and only if $\underline{R}(X) = \overline{R}(X)$. Else, X is rough with respect to R if and only if $\underline{R}(X) \neq \overline{R}(X)$. $\underline{R}$ and $\overline{R}$ are called, respectively, the lower approximation operator and upper approximation operator with respect to R .

This model is Pawlak rough set model. If $[x]_R \subseteq X$, we usually say that x is precisely supporting the concept X with respect to R . Correspondingly, if $[x]_R \cap X \neq \emptyset$, it is said that x is possibly supporting the concept X with respect to R .

Theorem 2.1([3,14,15]) Let U be the universe. $R \subseteq U \times U$ is an equivalence relation on the universe. For any $X, Y \subseteq U$, the following properties of lower and upper approximation operators hold.

- (1) $\underline{R}(X) \subseteq X \subseteq \overline{R}(X)$;
- (2a) $\underline{R}(\sim X) = \sim \overline{R}(X)$;
- (2b) $\overline{R}(\sim X) = \sim \underline{R}(X)$;
- (3a) $\underline{R}(\emptyset) = \overline{R}(\emptyset) = \emptyset$;
- (3b) $\underline{R}(U) = \overline{R}(U) = U$;
- (4a) $X \subseteq Y \Rightarrow \underline{R}(X) \subseteq \underline{R}(Y)$;
- (4b) $X \subseteq Y \Rightarrow \overline{R}(X) \subseteq \overline{R}(Y)$;
- (5a) $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$;
- (5b) $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$;
- (6a) $\underline{R}(X \cup Y) \subseteq \underline{R}(X) \cup \underline{R}(Y)$;
- (6b) $\overline{R}(X \cap Y) \supseteq \overline{R}(X) \cap \overline{R}(Y)$.

Classes are the useful descriptions to characterize concepts in rough set theory. Possible descriptions