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A Class of Quasi-Quartic Trigonometric BÉZier Curves and Surfaces

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Abstract. A new kind of quasi-quartic trigonometric polynomial base functions with a shape parameter λ over the space Ω=span {1, sin*t*, cos*t*, sint2*t*, cos2*t*} is presented, and the corresponding quasi-quartic trigonometric Bézier curves and surfaces are defined by the introduced base functions. The quasi-quartic trigonometric Bézier curves inherit most of properties similar to those of quartic Bézier curves, and can be adjusted easily by using the shape parameter λ. The jointing conditions of two pieces of curves with G^2 and C^4 continuity are discussed. With the shape parameter chosen properly, the defined curves can express exactly any plane curves or space curves defined by parametric equation based on{1, sin*t*, cos*t*, sint2*t*, cos2*t*} and circular helix with high degree of accuracy without using rational form. The corresponding tensor product surfaces can also represent precisely some quadratic surfaces, such as sphere, paraboloid, cylindrical surfaces, and some complex surfaces. The relationship between quasi-quartic trigonometric Bézier curves and quartic Bézier curves is also discussed, the larger is parameter λ, and the more approach is the quasi-quartic trigonometric Bézier curve to the control polygon. Examples are given to illustrate that the curves and surfaces can be used as an efficient new model for geometric design in the fields of CAGD.

Keywords: Bézier curves and surfaces, trigonometric polynomial, quasi-quartic, shape parameter, G^2 and C^4 continuity

1. Introduction

In Computer Aided Geometric Design (CAGD), lower order Bézier curves and B-spline curves have become the common tools for constructing free form curves and surfaces [1, 2]. But they cannot represent exactly some quadratic curves such as the circular arcs, parabolas, spheres, cylinders and the other conic curves and surfaces. Although rational Bézier curves and NURBS curves can construct some analytic curves and surfaces, such as conic curves and revolution surfaces, there are some defects because of theirs rational style, such as complexity of computing derivation and quadrature, the weights of selecting not easy to control [3, 4].

In recent years, people have gained interest in trigonometric polynomial curve spline and have started to search the represent method to construct curves and surfaces on the space of trigonometric functions, of which in [5] the famous C-curves is obtained based on {1, *t*, sin*t*, cos*t*}, quadratic and cubic trigonometric polynomial curves with two shape parameters are given respectively in [6] and [7], a group of T-Bézier curves with features of Bézier curves is proposed in [8], the adjustable quadratic trigonometric Bézier curves with a shape parameter are presented in [9,10]. These existing trigonometric curves have similar properties to polynomial curves.

In this paper, we present a class of new different trigonometric polynomial basis functions with a parameter based on the space Ω=span {1, sin*t*, cos*t*, sint2*t*, cos2*t*}, and the corresponding curves and tensor product surfaces named quasi-quartic trigonometric Bézier curves and surfaces are constructed based on the introduced basis functions. The quasi-quartic trigonometric Bézier curves not only inherit most of the similar

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properties to quartic Bézier curves, but also can express any plane curves or space curves defined by parametric equation based on {1, sin*t*, cos*t*, sint2*t*, cos2*t*} including some quadratic curves such as the circular arcs, parabolas, cardioid exactly and circular helix with high degree of accuracy under the appropriate conditions.

The rest of this paper is organized as follows. Section 2 defines the quasi-quartic trigonometric polynomial base functions and the corresponding curves and surfaces, theirs propositions are discussed. In section 3, we discussed the continuity conditions of quasi-quartic trigonometric Bézier curves. In section 4, we show the representations of some curves. Besides, some examples of shape modeling by using the quasi-quartic trigonometric Bézier surfaces are presented also. We devote Section 5 to giving the relationship between quasi-quartic trigonometric Bézier curves and quartic Bézier curves. The conclusions are given in section 6.

2. Construction and related properties of quasi-quartic trigonometric Bézier curves and surfaces

Definition 1 For $t \in [0, \frac{\pi}{2}]$, $b_{0,4}(t)$, $b_{1,4}(t)$, $b_{2,4}(t)$, $b_{3,4}(t)$ and $b_{4,4}(t)$ are called quasi-quartic trigonometric polynomial base functions with a shape parameter λ which can be defined to be

$$\begin{cases} b_{0,4} = (1+\frac{\lambda}{2}) - (1+\lambda)\sin t - \frac{\lambda}{2}\cos 2t \\ b_{1,4} = (1+\lambda)(-\frac{3}{2} + 2\sin t + \cos t - \frac{1}{2}\sin 2t + \frac{1}{2}\cos 2t) \\ b_{2,4} = 2(1+\lambda)(1 - \sin t - \cos t + \frac{1}{2}\sin 2t) \\ b_{3,4} = (1+\lambda)(-\frac{3}{2} + \sin t + 2\cos t - \frac{1}{2}\sin 2t - \frac{1}{2}\cos 2t) \\ b_{4,4} = (1+\frac{\lambda}{2}) - (1+\lambda)\cos t + \frac{\lambda}{2}\cos 2t \end{cases} \tag{1}$$

where $-1 \le \lambda \le 1.5$.

From Eq. (1), it is easy to check that

1) Weight property: $b_{0,4}(t) + b_{1,4}(t) + b_{2,4}(t) + b_{3,4}(t) + b_{4,4}(t) \equiv 1$;

2) Symmetry: $b_{i,4}(\frac{\pi}{2} - t) = b_{4-i,4}(t)$, $i = 0,1,2,3,4$

3) Nonnegative property: when $-1 \le \lambda \le 1$, $b_{i,4}(t) \ge 0$ $(i = 0,1,2,3,4)$.

The above results show that the quasi-quartic trigonometric polynomial base functions have the most of properties similar to quartic Bernstein basis functions.

Definition 2 Let $\boldsymbol{P}_0$, $\boldsymbol{P}_1$, $\boldsymbol{P}_2$, $\boldsymbol{P}_3$ and $\boldsymbol{P}_4$ be given control points, the following curves are called quasi-quartic trigonometric Bézier curve,

$$\boldsymbol{B}(t) = \sum_{j=0}^{4} b_{j,4}(t)\boldsymbol{P}_j \tag{2}$$

where $b_{j,4}(t)$ $(j = 0,1,2,3,4)$ are quasi-quartic trigonometric polynomial base functions.

Let [$\boldsymbol{t}$] = [1 sin*t* cos*t* sint2*t* cos2*t*], [$\boldsymbol{P}$] = [$\boldsymbol{P}_0$ $\boldsymbol{P}_1$ $\boldsymbol{P}_2$ $\boldsymbol{P}_3$ $\boldsymbol{P}_4$] and

$$[\boldsymbol{b}] = \begin{bmatrix} 1+\frac{\lambda}{2} & -\frac{3(1+\lambda)}{2} & 2(1+\lambda) & -\frac{3(1+\lambda)}{2} & 1+\frac{\lambda}{2} \\ -(1+\lambda) & 2(1+\lambda) & -2(1+\lambda) & 1+\lambda & 0 \\ 0 & 1+\lambda & -2(1+\lambda) & 2(1+\lambda) & -(1+\lambda) \\ 0 & -\frac{1+\lambda}{2} & 1+\lambda & -\frac{1+\lambda}{2} & 0 \\ -\frac{\lambda}{2} & \frac{1+\lambda}{2} & 0 & -\frac{1+\lambda}{2} & \frac{\lambda}{2} \end{bmatrix}$$