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Quartic B-Spline Collocation Method for Solving One-Dimensional Hyperbolic Telegraph Equation

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Abstract. In this paper, we use a numerical method based on B-spline function and collocation method to solve second-order linear hyperbolic telegraph equation. The scheme works in a similar fashion as finite difference methods. The results of numerical experiments are presented, and are compared with analytical solutions to confirm the good accuracy of the presented scheme.

Keywords: B-spline, collocation method, second-order hyperbolic telegraph equation, difference schemes.

1. Introduction

We consider the second-order linear hyperbolic telegraph equation in one-space dimension, given by

$$\frac{\partial^2 u}{\partial t^2}+2\alpha\frac{\partial u}{\partial t}+\beta^2 u=\frac{\partial^2 u}{\partial x^2}+f(x,t),\ a\le x\le b,\ \ 0\le t\le T, \tag{1}$$

subject to initial conditions

$$u(x,0)=f_0(x),\quad a\le x\le b, \tag{2}$$

$$\frac{\partial u(x,0)}{\partial t}=f_1(x),\quad a\le x\le b, \tag{3}$$

and Dirichlet boundary conditions

$$u(a,t)=g_0(t),\quad u(b,t)=g_1(t),\quad t\ge 0, \tag{4}$$

where α and β are known constant coefficients. We assume that $f_0(x), f_1(x)$ and their derivatives are continuous functions of x, and $g_i(t), i=0,1,$ and their derivatives are continuous functions of t. Both the electric voltage and the current in a double conductor, satisfy the telegraph equation, where x is distance and t is time. For $\alpha>0$ and $\beta=0$, Eq. (1) represents a damped wave equation and for $\alpha>\beta>0$, it is called telegraph equation.

The hyperbolic partial differential equations model the vibrations of structures (e.g. buildings, beams and machines) and are the basis for fundamental equations of atomic physics. Equations of the form Eq. (1) arise in the study of propagation of electrical signals in a cable of transmission line and wave phenomena. Interaction between convection and diffusion or reciprocal action of reaction and diffusion describes a number of nonlinear phenomena in physical, chemical and biological process [1]-[4]. In fact the telegraph equation is more suitable than ordinary diffusion equation in modeling reaction diffusion for such branches of sciences. For example biologists encounter these equations in the study of pulsate blood flow in arteries and in one- dimensional random motion of bugs along a hedge [5]. Also the propagation of acoustic waves in Darcy-type porous media [6], and parallel flows of viscous Maxwell fluids [7] are just some of the phenomena governed [8]-[9] by Eq. (1).

The theory of spline functions is very active field of approximation theory, boundary value problems and partial differential equations, when numerical aspects are considered. Among the various classes of splines,

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the polynomial spline has been received the greatest attention primarily because it admits a basis of B-splines [10]-[14] which can be accurately and efficiently computed. As the piecewise polynomial, B-splines have also become a fundamental tool for numerical methods to get the solution of the differential equations. In this paper, numerical solution of the hyperbolic telegraph equation by using the quartic B-spline collocation scheme is proposed. The collocation method together with B-spline approximations represents an economical alternative since it only requires the evaluation of the unknown parameters at the grid points. As is known, the success of the B-spline collocation method is dependent on the choice of B-spline basis. The quartic B-spline basis has been used to build up the approximation solutions for some differential equations. For instance see [15]-[20].

The layout of the article is as follows. In Section 2, we show that how we use the B-spline collocation method to approximate the solution of the hyperbolic telegraph equation. To demonstrate the efficiency of the proposed method, numerical experiments are carried out for several test problems and results are given in section 3. Finally the conclusion is given in the last Section. Finally some references are introduced at the end. Note that we have computed the numerical results by Matlab programming.

2. Quartic B-spline collocation method

Let be a uniform partition of an interval $[a,b]$ as follows $a = x_0 < x_1 < ... < x_N = b$ where $h = x_{j+1} - x_j$, $j = 0,1,..., N-1$. The quartic B-splines are defined upon an increasing set of $N+1$ knots over the problem domain plus 8 additional knots outside the problem domain 8 additional knots are positioned as

$$x_{-4} < x_{-3} < x_{-2} < x_{-1} < x_0 \text{ and } x_N < x_{N+1} < x_{N+2} < x_{N+3} < x_{N+4}.$$

The set of quartic B-spline $\{Q_{-2}, Q_{-1},..., Q_{N+1}\}$ form a basis over the problem domain $[a,b]$ [12].

Let $Q_m(x), m = -2,-1,..., N+1,$

$$Q_m(x) = \frac{1}{h^4}\begin{cases} (x - x_{m-2})^4, & x \in [x_{m-2}, x_{m-1}], \\ (x - x_{m-2})^4 - 5(x - x_{m-1})^4, & x \in [x_{m-1}, x_m], \\ (x - x_{m-2})^4 - 5(x - x_{m-1})^4 + 10(x - x_m)^4, & x \in [x_m, x_{m+1}], \\ (x_{x+3} - x)^4 - 5(x_{x+2} - x)^4, & x \in [x_{m+1}, x_{m+2}], \\ (x_{x+3} - x)^4, & x \in [x_{m+2}, x_{m+3}], \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

be quartic B-splines, which vanish outside interval. Each quartic B-spline cover five elements so that an element is covered by five quartic B-splines.

Now the solution of the problem is considered as follows:

$$U_N(x,t) = \sum_{m=-2}^{N+1} \delta_m(t) Q_m(x), \tag{6}$$

where $\delta_m, m = -2,..., N+1$ are unknown time dependent quantities to be determined from boundary conditions and the initial conditions. The values of $Q_m(x)$ and its derivatives $Q_m^{'}(x)$, $Q_m^{''}(x)$ and $Q_m^{'''}(x)$ at the knots are given in Table 1.

Table 1: The values of $Q_m, Q_m^{'}, Q_m^{''}, Q_m^{'''}$.

x	x_{i-2}	x_{i-1}	x_i	x_{i+1}	x_{i+2}	x_{i+3}
Q	0	1	11	11	1	0
$Q_i^{'}$	0	$-4/h$	$-12/h$	$12/h$	$4/h$	0
$Q_i^{''}$	0	$12/h^2$	$-12/h^2$	$-12/h^2$	$12/h^2$	0
$Q_i^{'''}$	0	$-24/h^3$	$72/h^3$	$-72/h^3$	$24/h^3$	0

For every x by using the Taylor expansion in the time direction, using the notation $u_i = u(x, t_i)$ where