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A Sinc-Collocation Method for Second-Order Boundary Value Problems of Nonlinear Integro-Differential Equation

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Abstract. The sinc-collocation method is presented for solving second-order boundary value problems of nonlinear integro-differential equation. The method is effective for approximation in the case of the presenceof end-point singularities. Some properties of the sinc-collocation method required for our subsequent development are given and are utilized to reduce the computation of solution of the second-order boundary value problems of nonlinear integro-differential equation to some algebraic equations. Some numerical results are also given to demonstrate the validity and applicability of the presented technique.

Keywords: Sinc function, Collocation method, Boundary value problems, Second-order, Nonlinear integro-differential equation.

1. Introduction

Boundary value problems for integro-differential equations are important because they have many applications in the study of physical, biological and chemical phenomena [1]. Liz and Nieto [2], study a two point boundary value problem for a nonlinear second order integro-differential equation of Fredholm type by using upper and lower solutions. In [1], an iterative method is presented to solve a class of boundary value problems for second-order integro-differential equation in the reproducing kernel space. For linear and nonlinear second order Fredholm integro-differential equations, semiorthogonal spline wavelets was developed in [3] and Chebyshev finite difference method was discussed in [4]. Also in [5], Saadatmandi and Dehghan applied the Legendre polynomials for the solution of the linear Fredholm integro-differential-difference equation of high order.

In this paper, a sinc-collocation procedure is developed for the numerical solution second-order boundary value problems of nonlinear integro-differential equation of the form:

$$u''(x)+p(x)u'(x)+q(x)u(x)+\lambda_1\int_a^x k_1(x,t)u(t)dt+\lambda_2\int_\Gamma k_2(x,t)u(t)dt=f\big(x,u(x)\big), \qquad (1)$$

$$x,t\in\Gamma=[a,b], \quad u(a)=\alpha, \qquad u(b)=\beta,$$

where the parameters λ_1, λ_2, the kernels $k_1(x,t)$, $k_2(x,t)$, the functions $p(x), q(x)$ are given and $f(x,u(x))$ is nonlinear in $u(x)$, where $u(x)$ is the unknown function to be determined. There has been a great deal of research work on the existence of solutions for boundary value problems, for instance see [6, 7, 8].

Sinc methods have increasingly been recognized as powerful tools for problems in applied physics and engineering [9, 10]. The sinc-collocation method is a simple method with high accuracy for solving a large variety of nonlinear problems. In Reference [11], the sinc-collocation method is presented for solving boundary value problems for nonlinear third-order differential equations. Authors of [12], used the sinc-collocation method for solving a nonlinear system of second-order boundary value problems. Mohsen and El-Gamel [13], used the sinc-collocation method for solving the linear integro-differential equations of the Fredholm type. Also in [14], the sinc-collocation is presented for solving linear and nonlinear Volterra integral and integro-differential equations. In [15, 16], the sinc-collocation is used for the numerical solution Fredholm and Volterra integro-differential equations. Also sinc-collocation method is used for solving of a system of nonlinear second-order integro-differential equations with boundary conditions of the Fredholm and Volterra types [17]. We also refer the interested reader to [18, 19, 20, 21, 22, 23] for more research works on sinc methods.

The main purpose of the present paper is to develop methods for numerical solution of the second-order boundary value problems of nonlinear integro-differential equation (1). Our method consists of reducing the solution of (1) to a set of algebraic equations. The properties of sinc function are then utilized to evaluate the unknown coefficients. The organization of the rest of this article is as follows. In Section 2, we review some of the main properties of sinc function that are necessary for our subsequent development. In Section 3, we illustrate how the sinc method may be used to replace Eq. (1) by an explicit system of nonlinear algebraic equations. Section 4, presents appropriate techniques to treat no homogeneous boundary conditions. In Section 5, some numerical results are given to clarify the method.

2. Sinc function properties

Sinc function properties are discussed thoroughly in [9, 10]. In this section an overview of the formulation of the sinc function required for our subsequent development is presented. The sinc function is defined on the whole real line, $-\infty < x < \infty$, by

$$Sinc(x) = \begin{cases} \dfrac{\sin(\pi x)}{\pi x}, & x \neq 0, \\ 1, & x = 0. \end{cases} \tag{2}$$

For any $h > 0$, the translated sinc functions with evenly spaced nodes are given by

$$S(j,h)(x) = Sinc\left(\frac{x - jh}{h}\right) = \begin{cases} \dfrac{\sin\left[\frac{\pi}{h}(x - jh)\right]}{\frac{\pi}{h}(x - jh)}, & x \neq jh \\ 1, & x = jh \end{cases} \tag{3}$$

which are called the jth sinc functions. The sinc function form for the interpolating point $x_k = kh$ is given by

$$S(j,h)(kh) = \delta_{jk} = \begin{cases} 1, & j = k, \\ 0, & j \neq k. \end{cases} \tag{4}$$

If u is defined on the real line, then for $h > 0$ the series

$$C(u,h)(x) = \sum_{j=-\infty}^{\infty} u(jh) Sinc\left(\frac{x - jh}{h}\right), \tag{5}$$

is called the Wittaker cardinal expansion of u, whenever this series converges [9,10]. But in practice we need to use some specific numbers of terms in the above series, such as $j = -N, \ldots, N$, where N is the number of sinc grid points. They are based in the infinite strip D_S in the complex plane

$$D_S = \left\{ w = u + iv : |v| < d \leq \frac{\pi}{2} \right\}. \tag{6}$$

To construct an approximation on the interval (a, b), we consider the conformal map

$$\phi(z) = Ln\left(\frac{z - a}{b - z}\right). \tag{7}$$

The map carries the eye-shaped region

$$D_E = \left\{ z \in \mathbb{C} : \left| arg\left(\frac{z - a}{b - z}\right) \right| < d \leq \frac{\pi}{2} \right\}. \tag{8}$$

For the sinc method, the basis functions on the interval (a, b) for $z \in D_E$ are derived from the composite translated sinc functions,