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A Second Kind Chebyshev Polynomial Approach for the Wave Equation Subject to an Integral Conservation Condition

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Abstract. The main purpose of this article is to present an approximate solution for the one dimensional wave equation subject to an integral conservation condition in terms of second kind Chebyshev polynomials. The operational matrices of integration and derivation are introduced and utilized to reduce the wave equation and the conditions into the matrix equations which correspond to a system of linear algebraic equations with unknown Chebyshev coefficients. Finally, some examples are presented to illustrate the applicability of the method.

Keywords: Wave equation, Non-local condition, Second kind Chebyshev polynomials, Operational matrix, Matrix form.

1. Introduction

Hyperbolic partial differential equations with an integral condition serve as models in many branches of physics and technology. There are many papers that deal with the numerical solution of the parabolic equation with integral conditions [1, 4, 5, 6, 7, 11, 13]. The present work focuses on the one-dimensional wave equation with the non-local boundary condition.

In this paper, we consider the following one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = F(x,t), \quad 0 \le x \le \ell, \ 0 < t \le T. \tag{1}$$

with initial conditions

$$u(x,0) = f_1(x), \ 0 \le x \le \ell, \tag{2}$$

and

$$u_t(x,0) = f_2(x), \ 0 \le x \le \ell, \tag{3}$$

and Dirichlet boundary condition

$$u(0,t) = g_1(t), \ 0 < t \le T, \tag{4}$$

and the non-local condition

$$\int_0^\ell u(x,t)dx = g_2(t), \ 0 < t \le T, \tag{5}$$

where F, f_1, f_2, g_1 and g_2 are known functions.

The existence and uniqueness of the solution of the problem (1)--(5) are discussed in [3]. Dehghan [8] presented finite difference schemes for the numerical solution of problem (1)--(5). In [15] the shifted Legendre Tau technique was developed for the solution of the studied model. Author of [2] developed a numerical technique based on an integro- differential equation and local interpolating functions for solving the one-dimensional wave equation subject to a non-local conservation condition and suitably prescribed initial-boundary conditions. Authors of [14] combined finite difference and spectral methods to solve the one-dimensional wave equation with an integral condition. In [9] variational iteration method was applied for

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solving the wave equation subject to an integral conservation condition. Authors of [10] presented a meshless method for numerical solution of problem (1)--(5). Also in [16] the method of lines was developed for the solution of the discussed problem.

Orthogonal functions have been used to solve various problems. The main characteristic of this technique is that it reduces problem to those of solving a system of algebraic equations thus greatly simplifying the problem. In the present paper, the numerical solution of the problem (1)--(5) is computed by using two variable shifted second kind Chebyshev orthogonal functions.

The paper is organized as follows: In Section 2, basic properties of the second kind Chebyshev polynomials are presented. In Section 3, we discuss how to approximate functions in terms of second kind Chebyshev polynomials and introduce operational matrices of integration and derivation. In section 4, we give an approximate solution for (1)--(5). Numerical examples are given in Section 5 to illustrate the accuracy of our method. Finally, concluding remarks are given in Section 6.

2. Properties of the Second Kind Chebyshev Polynomials

Second kind Chebyshev polynomials are total orthogonal basis for $L^2[-1,1]$ and can be determined with the aid of the following recursive formula [12].

$$\begin{aligned} &U_0(t) = 1, \\ &U_1(t) = 2t, \\ &U_n(t) = 2tU_{n-1}(t) - U_{n-2}(t), n \geq 2, -1 \leq t \leq 1. \end{aligned}$$

For the case that the interval is not $[-1,1]$, say $[a,b]$, we can use the linear transformation

$$t' = \frac{2t-a-b}{b-a},$$

to transform the domain into $[-1,1]$.

The orthogonality property is as follows:

$$\int_{-1}^{1} \omega(t) U_i(t) U_j(t) dt = \begin{cases} \dfrac{\pi}{2}, & i = j, \\ 0, & i \neq j, \end{cases}$$

where $\omega(t) = \sqrt{1-t^2}$ is the weight function.

Some properties of the second kind Chebyshev polynomials are as follows:

$$\int_{-1}^{1} U_n(t) dt = \begin{cases} \dfrac{2}{n+1}, & n = 2k, \\ 0, & n = 2k+1, \end{cases} \tag{6}$$

$$\int_{-1}^{t} U_n(t')dt' = \frac{1}{n+1}[(-1)^n U_0(t) - \frac{1}{2}U_{n-1}(t) + \frac{1}{2}U_{n+1}(t)],\ U_{-1}(t) = 0, \tag{7}$$

$$U_n'(t) = 2\sum_{i=0}^{[n/2]} (n-2i) U_{n-2i-1}(t), \tag{8}$$

$$U_n(-1) = (-1)^n (n+1). \tag{9}$$

3. Function Approximation

3.1. Approximation of one-variable and two-variable functions

A function $y(t)$ defined over $[0,T]$ may be expanded by the shifted second kind Chebyshev functions as

$$y(t) = \sum_{j=0}^{\infty} a_j U_j(\frac{2}{T}t - 1) = \sum_{j=0}^{\infty} a_j \varphi_j(t), \tag{10}$$