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A Comparison of Numerical Solutions of Eleventh Order Two-point Boundary Value Problems

Amjad Hussain [1+], Syed Tauseef Mohyud-Din [2] and Ahmet Yildirim [3]

[1] Department of Mathematics, Zhejiang University, Hangzhou 310027, PR China
[2] Department of Mathematics, HITEC University, Taxila Cantt Pakistan
[3] 4146 SK No 16 Zeytinalani Mah. 35440 Urla-Izmir, Turkey

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Abstract. This study considers an application of differential transform method (DTM) to solving linear and nonlinear eleventh order two-point boundary value problems discussed in [S.S. Siddiqi, G. Akram and I. Zulfiqar. Solution of eleventh order boundary value problems using variational iteration technique. *European Journal of Scientific Research*. 2009, **30**: 505-525]. It is shown that the proposed method yields excellent approximations at minimum computational cost, whereas, variational iteration method is computationally expensive for such problems. Numerical results explicitly reveal the complete reliability and efficiency of the proposed algorithm.

Keywords: Eleventh order boundary value problems, Differential Transform Method, Comparison.

1. Introduction

Differential transform algorithm for obtaining approximate series solution to the differential equations is fairly well known. The technique was proposed by Zhou [1] in 1986, to solve linear and nonlinear problems arising in electrical circuits. Since then it has been extensively used by many others to solve a variety of problems, like for example Blasius boundary layer flow [2], Non-Newtonian fluid flow between two parallel plates [3], Nonlinear equations arising in heat transfer [4], vibration analysis of pipes covering fluids [5], free vibration analysis of a centrifugally stiffened beam [6], vibration of an elastic beam supported on elastic soil [7], Eigen-value problems [8-10], and so on. At the present time, DTM is considered sufficiently accurate for most scientific applications. Current research is aimed at evaluating this technique for eleventh order boundary value problems.

The boundary value problems of higher order have been investigated due to their mathematical importance and the potential for applications in numerous fields of science and engineering. Agarwal [11] presented the theorems stating the conditions for the existence and uniqueness of solutions of such BVPs, while no numerical methods are contained therein. Eleventh order boundary value problem have not taken much attention in the literature. Siddiqi et al. [12] applied variational iteration method (VIM) to obtain approximations for such problems there by converting the original problem into a system of integral equations. Although, VIM works well, but for technical reasons the calculations are more voluminous. To solve these problems at a minimum computational cost we implement differential transform method. It is found that the method is versatile and numerically programmable.

In this paper, we consider eleventh order boundary value problem in the form:

$$y^{(11)}(x) = f(x, y), \qquad x \in [a, b] \tag{1}$$

subject to the boundary conditions

+ Corresponding author. Tel.: +86-571-87994844.
E-mail address: ahabbasi123@yahoo.com; ahabbasi123@hotmail.com.

$$\left.\begin{array}{ll} y(a)=A_1, & y(b)=B_1, \\ y^{(1)}(a)=A_2, & y^{(1)}(b)=B_2, \\ y^{(2)}(a)=A_3, & y^{(2)}(b)=B_3, \\ y^{(3)}(a)=A_4, & y^{(3)}(b)=B_4, \\ y^{(4)}(a)=A_5, & y^{(4)}(b)=B_5, \\ y^{(5)}(a)=C_1. & \end{array}\right\} \tag{2}$$

where $f(x,y)$ is assumed to be real and as many times differentiable as required for $x \in [a,b]$ and A_i, $B_i; i=1,\ldots,5$ and C_1 are real constants. In the next Section, we will tabulate some rules to implement differential transform method.

2. The Differential Transform Method

With reference to the articles [13-15], the basic definitions of differential transform are as follows

Definition 2.1 if $f(t)$ is analytic in the time domain T, then it can be differentiated continuously with respect to time t

$$\Phi(t,k)=\frac{d^k f(t)}{dt^k}, \qquad \text{for all } t \in T \tag{3}$$

for $t=t_i, \Phi(t,k)=\Phi(t_i,k)$ where k belongs to the set of non-negative integers, denoted as K-domain. Therefore (3) can be written as

$$F(k)=\Phi(t_i,k)=\left[\frac{d^k f(t)}{dt^k}\right]_{t=t_i}, \qquad \text{for all } k \in K \tag{4}$$

where $F(k)$ is called spectrum of $f(t)$ at $t=t_i$ in the K-domain.

Definition 2.2 if $f(t)$ can be expressed by Taylor series, then $f(t)$ can be represented as

$$f(t)=\sum_{k=0}^{\infty}\left[\frac{(t-t_i)^k}{k!}\right]F(k). \tag{5}$$

Equation (5) is called inverse differential transform of $F(k)$.

Using the differential transform, a differential equation in the domain of interest can be transformed to an algebraic equation in the K-domain and then $f(t)$ can be obtained by finite-term Taylor's series plus a remainder, as

$$f(t)=\sum_{k=0}^{n}\left[\frac{(t-t_i)^k}{k!}\right]F(k)+R_{n+1}(t). \tag{6}$$

In order to speed up the convergence rate and the accuracy of calculation, the entire domain of t needs to be split into sub-domains [13, 17]. The fundamental operations performed by differential transform can readily be obtained and are listed in Table 1.