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A modified (G'/G)- expansion method and its application for finding hyperbolic, trigonometric and rational function solutions of nonlinear evolution equations

Elsayed M. E. Zayed [1 +]

[1] Mathematics Department, Faculty of Science, Zagazig University, Zagazig, Egypt

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Abstract. A modified (G'/G)- expansion method is proposed to construct hyperbolic, trigonometric and rational function solutions of nonlinear evolution equations which can be thought of as the generalization of the (G'/G)- expansion method given recently by M.Wang et al . To illustrate the validity and advantages of this method, the (1+1)-dimensional Hirota-Ramani equation and the (2+1)-dimensional Breaking soliton equation are considered and more general traveling wave solutions are obtained. It is shown that the proposed method provides a more general powerful mathematical tool for solving nonlinear evolution equations in mathematical physics.

Keywords: Nonlinear evolution equations, modified (G'/G)-expansion method, hyperbolic Function solutions, Trigonometric Function solutions, Rational function solutions.

1. Introduction

Nonlinear evolution equations are often presented to describe the motion of isolated waves, localized in a small part of space, in many fields such as hydrodynamics, plasma physics, and nonlinear optics. Seeking exact solutions of these equations plays an important role in the study of these nonlinear physical phenomena. In the past several decades, many effective methods for obtaining exact solutions of these equations have been presented,such as the inverse scattering method [2], the Hirota bilinear method [8,9], the Backlund transformation [8,18], the Painleve expansion method [12,13,14,22],the Sine-Cosine method [21,25], the Jacobi elliptic function method [15,16], the tanh-function method [6,23,27,31], the F-expansion method [32], the exp-function method [4,5,7], the (G'/G) -expansion method [3,10,11,17,19,20,24,26,28-30], the modified (G'/G) -expansion method [1,17,30] and so on. Wang et.el [20] introduced the (G'/G) -expansion method to look for traveling wave solutions of nonlinear evolution equations. This method is based on the assumption that these solutions can be expressed by a polynomial in (G'/G), and that $G = G(\xi)$ satisfies a second order linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{1}$$

where λ, μ are constants and $' = d/d\xi$, while $\xi = kx + \omega t$, and k , ω are constants. The degree of this polynomial can be determined by considering the homogeneous balance between the highest-order derivatives and nonlinear terms appearing in the given nonlinear evolution equations. The coefficients of this polynomial can be obtained by solving a set of algebraic equations resulted from the process of using the method. The present paper is motivated by the desire to propose a modified (G'/G)-expansion method for constructing more general exact solutions of nonlinear evolution equation. To illustrate the validity and advantages of the proposed method, we would like to employ it to solve the (1+1)- dimensional Hirota Ramani equation [1,9] and the (2+1)-dimensional Breaking soliton equation [26,29].

The rest of this paper is organized as follows: In Sec.2, we describe the modified (G'/G)-expansion method. In Sec.3, we use this method to solve the two nonlinear equations indicated above. In Sec.4, conclusions are given.

+ *E-mail address*: e.m.e.zayed@hotmail.com.

2. Description of the modified (G'/G)-expansion method

For a given nonlinear evolution equation in the form

$$P(u, u_x, u_y, u_t, u_{xx}, u_{xt}, \ldots), \tag{2}$$

where $u(x, y, t)$, we use the wave transformation

$$u(x, y, t) = u(\xi), \quad \xi = k_1 x + k_2 y + \omega t, \tag{3}$$

where k_1, k_2, ω are constants, then Eq.(1) is reduced into the ordinary differential equation

$$Q\left(u^{(r)}, u^{(r+1)}, \ldots\right) = 0, \tag{4}$$

where $u^{(r)} = \dfrac{d^r u}{d\xi^r}, r \geq 0$ and r is the least order of derivatives in the equation. Setting $u^{(r)} = V(\xi),$ where $V(\xi)$ is a new function of ξ, we further introduce the following anstaz :

$$u^{(r)}(\xi) = V(\xi) = \sum_{i=0}^{m} \alpha_i \left(\frac{G'}{G} + \frac{\lambda}{2}\right)^i, \; \alpha_m \neq 0, \tag{5}$$

where $G = G(\xi)$ satisfies Eq.(1) while $\alpha_i \, (i = 0,1,\ldots,m)$ are constants to be determined later.

To determine $u(\xi)$ explicitly, we take the following four steps :

Step 1. Determine the positive integer m in Eq. (5) by balancing the highest-order nonlinear terms and the highest-order derivatives, in Eq. (4).

Step 2. Substitute Eq.(5) along with Eq.(1) into Eq. (4) and collect all terms with the same powers of $\left(\frac{G'}{G} + \frac{\lambda}{2}\right)^i$, $i = (0,1,\ldots,m)$ together, thus the left-hand side of (4) is converted into a polynomial in $\left(\frac{G'}{G} + \frac{\lambda}{2}\right)^i$. Then set each coefficient of this polynomial to zero, to derive a set of algebraic for $\alpha_i, k_1, k_2, \omega, i = (0,1,\ldots,m)$.

Step 3. Solve these algebraic equations by use of Mathematica to find the values of $\alpha_i, k_1, k_2, \omega,$ $i = (0,1,\ldots,m)$.

Step 4. Use the results obtained in above steps to derive a series of fundamental solutions $V(\xi)$ of Eq.(4) depending on $\left(\frac{G'}{G} + \frac{\lambda}{2}\right)$. Since the solutions of Eq.(1) have been well known for us as follows:

(i) If $\lambda^2 - 4\mu > 0$, then

$$\left(\frac{G'}{G} + \frac{\lambda}{2}\right) = \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\left[\frac{c_1 \sinh\left(\frac{\xi}{2}\sqrt{\lambda^2 - 4\mu}\right) + c_2 \cosh\left(\frac{\xi}{2}\sqrt{\lambda^2 - 4\mu}\right)}{c_1 \cosh\left(\frac{\xi}{2}\sqrt{\lambda^2 - 4\mu}\right) + c_2 \sinh\left(\frac{\xi}{2}\sqrt{\lambda^2 - 4\mu}\right)}\right]. \tag{6}$$

(ii) If $\lambda^2 - 4\mu < 0$, then