ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 8, No. 1, 2013, *pp*. 041-054

An Alternative Approach for PDE-Constrained Optimization via Genetic Algorithm

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(*Received September 10, 2012, accepted November 22, 2012*)

Abstract. This paper deals with an alternative approach for solving the PDE-constrained optimization problems. For this purpose, the problem has been discretized with the help of finite difference method. Then the reduced problem has been solved by advanced real coded genetic algorithm with ranking selection, whole-arithmetic crossover and non uniform mutation. The proposed approach has been illustrated with a numerical example. Finally, to test the performance of the algorithm, sensitivity analyses have been performed on objective function values with respect to different parameters of genetic algorithm.

Keywords: *PDE-Constrained optimization, optimal control, genetic algorithm.*

1. Introduction

PDE-Constrained optimization refers to the optimization of systems governed by partial differential equations. This means that the constraints of the optimization problems are partial differential equations. Most of the physical problems are modeled mathematically by a system of partial differential equations. Due to the complexity of the partial differential equations, analytical solutions to these equations do not exist in general. Hence to solve these partial differential equations, there is only one way to find the approximate numerical solutions.

Finding the solutions of constrained optimization problems is a challenging as well as demanding task in the competitive market situations due to globalization of market economy. Parallel research has been going on in PDE-simulation and numerical optimization. Currently there is a growing tendency of cooperation and collaboration among these two communities in order to achieve the best possible results in practical applications.

To the best of our knowledge, very few researchers have solved this type of problems in the last few years. In this connection, one may refer to the recent works of Hazra [1, 2], Hazra and Schulz [3], Hazra et al. [4], Biros and Ghattas [5], Emilio et al. [6], Griesse and Vexler [7], Rees, Stoll and Wathen [8], Rees, Dollar and Wathen [9] along with others.

Hazra [1] proposed simultaneous pseudo-timestepping as an efficient method for aerodynamic shape optimization. In this method, instead of solving the necessary conditions by iterative techniques, pseudo-time embedded nonstationary system is integrated in time until a steady state is reached. Hazra and Schulz [3] developed a method for the optimization problems with PDE constraints. This method can be viewed as a reduced SQP method in the sense that it uses a preconditioner derived from that method. The reduced Hessian in the preconditioner is approximated by a pseudo-differential operator. Hazra et al. [4] proposed a new method based on simultaneous pseudo-timestepping for solving aerodynamic shape optimization problem. The preconditioned pseudo-stationary state, costate and design equations are integrated simultaneously in time until a steady state is reached. The preconditioner used in this study is motivated by a continuous re-interpretation of reduced SQP methods. Biros and Ghattas [5] proposed a method for the steady-state PDE-constrained optimization, based on the idea of using a full space Newton solver combined with an approximate reduced space quasi-Newton SQP preconditioner. The basic components of this method are Newton solution of the first-order optimality conditions that characterize stationarity of the Lagrange function. Emilio et al. [6] solved the shape optimization problem in ship hydrodynamics using computational fluid dynamics. Griesse and Vexler [7] considered the efficient computation of derivatives of a functional which depends on the solution of a PDE-constrained optimization problem with inequality constraints. They derived the conditions under with the quantity of interest possesses first and second order derivatives with respect to the perturbation parameters. They developed an algorithm for the efficient evaluation of these derivatives with considerable savings over a direct approach, especially in case of high dimensional parameter spaces. To test the efficiency of the algorithm, numerical experiments involving a parameter identification problem for Navier-Stokes flow and an optimal control problem for a reaction-diffusion system have been presented. Hazra [2] proposed a method, called multigrid one-shot method, for solving state constrained aerodynamic shape optimization problems. Here multigrid strategy has been used to reduce the total number of iterations. Rees, Stoll and Wathen [8] illustrated how all-at-once methods could be employed to solve the problems from PDE-constrained optimization problems. In particular, they showed that both the problems with and without constraints lead to linear systems in saddle point form and also presented efficient preconditioning strategies for both problems. Rees, Dollar and Wathen [9] considered simple PDE-constrained optimization problem, viz. distributed control problems in which the constraint is either 2 or 3 - dimensional Poission PDE. Using discretization, the given constraints are converted to linear system with large dimension. To solve the systems, they introduced two optimal preconditioners for those systems which lead to convergence of symmetric Krylov subspace iterative methods in a number of iterations which does not increase with the dimension of the discrete problem. These preconditioners are block structured and involve standard multigrid cycles. The optimality of the preconditioned iterative solver is proved theoretically and verified computationally in several test cases.

The earlier mentioned methods are gradient based methods. However, these methods have some limitations. Among these limitations, one is that the traditional non-liner optimization methods very often stuck to the local optimum. To overcome these limitations, generally, soft computing methods like Genetic Algorithm, Simulated Annealing and Tabu search, are used for solving decision-making problems. Among these methods genetic algorithm is very popular. It is a well-known computerized stochastic search method based on the evolutionary principle "survival of the fittest" of Charles Darwin and natural genetics. It is executed iteratively on the set of real / binary coded solution called population. In each iteration, three basic genetic operations i.e., selection, crossover and mutation are performed. The concept of this algorithm was conceived by Prof. John Holland [10], University of Michigan, Ann Arbar in the year 1975. Thereafter, he and his students contributed much of the development of the subject. Goldberg [11] first popularized this subject by writing a text book. After that, several text books (Michalawicz [12], Gen and Chang [13], Sakawa [14], Eiben and Smith [15]) have been published in this area.

In this paper, an alternative approach has been proposed to solve the constrained optimization problem subject to the Poisson partial differential equations with Dirichlet boundary condition. In this approach, firstly the given problem has been discretized by finite difference method. Then the reduced problem has been solved by real coded advanced genetic algorithm. Next to illustrate the proposed approach, a numerical example has been solved. Finally, the performance of the proposed approach is tested by the sensitivity analysis on the best and mean objective function values with respect to the parameters of genetic algorithm.

2. The Problem

In this paper, we consider the optimization problem as follows:

$$\text{Minimize } J\left(y,u\right)=\frac{1}{2}\left\|y-\overline{y}\right\|_{L_2(\Omega)}^2+\frac{\beta}{2}\left\|u\right\|_{L_2(\Omega)}^2 \text{ , } \beta>0 \tag{1}$$