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Chebyshev Semi-iterative Method to Solve Fully Fuzzy linear Systems

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Abstract. In this paper, semi-iterative method is applied to find solution of the fully fuzzy linear systems. The convergence of this method is discussed in details. Furthermore, we show that in some situations that the existing methods such as Jacobi, Gauss-Seidel, JOR, SOR and are divergent, our proposed method is applicable. Finally, numerical computations are presented based on a particular linear system, which clearly show the reliability and efficiency of our algorithms

Keywords: iterative methods, semi-iterative methods, Chebyshev, fuzzy numbers, fuzzy arithmetic, fuzzy linear equations.

1. Introduction

Let us consider the following linear systems

$$Ax=b, \tag{1}$$

where $A \in R^{n \times n}, b, x \in R^n$. These method often occur in a wide variety of area including numerical differential equation, eigenvalue problems, economics models, design and computer analysis of circuits, power system networks, chemical engineering processes, physical and biological sciences ; see [1-12] and the references therein.

However, when the estimation of the system coefficients is imprecise and only some vague knowledge about the actual values of the parameters is available, it may be convenient to represent some or all of them with fuzzy numbers [13]. Fuzzy data is being used as a natural way to describe uncertain data. Fuzzy concept was introduced by Zadeh [13- 14]. We refer the reader to [15] for more information on fuzzy numbers and fuzzy arithmetic. Fuzzy systems are used to study a variety of problems including fuzzy metric spaces [16], fuzzy differential equations [17], particle physics [18- 19] , Game theory [20], optimization [21] and fuzzy linear systems[22-25].

Fuzzy number arithmetic is widely applied and useful in computation of linear system whose parameters are all or partially represented by fuzzy numbers. Dubois and Prade [26-27] investigated two definitions of a system of fuzzy linear equations, consisting of system of tolerance constraints and system of approximate equalities. The simplest method for finding a solution for this system is creating scenarios for the fuzzy system, which is a realization of fuzzy systems. Based on these actual scenarios, Buckley and Qu [28] extended several methods for this category and proved their approaches are not practicable, because infinite number of scenarios can be driven for a fully fuzzy linear system (FFLS). Friedman *et al*. [22] introduced a general model for solving a fuzzy n × n linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector. They used the parametric form of fuzzy numbers and replaced the original fuzzy n × n linear system by a crisp 2n × 2n linear system and studied duality in fuzzy linear systems $AX = BX+Y$ where A , B are real $n \times n$ matrix, the unknown vector X is vector consisting of n fuzzy numbers and the constant Y is vector consisting of n fuzzy numbers, in [29]. There are many other numerical methods for solving fuzzy linear systems such as Jacobi, Gauss-Seidel, Adomiam decomposition method and SOR iterative method [30-35]. In addition, another important kind of fuzzy linear systems are the fully fuzzy linear systems (FFLS) in which all the parameters are fuzzy numbers. Dehghan and Hashemi [36-37] proposed the Adomian decomposition method, and other iterative methods to find the positive fuzzy vector solution of $n \times n$ fully fuzzy linear system. Dehghan *et al.* [38] proposed some computational methods such as Cramer's rule, Gauss elimination method, *LU* decomposition method and linear programming approach for finding the approximated solution of FFLS. Nasseri *et al.* [39] used a certain decomposition methods of the coefficient matrix for solving fully fuzzy linear system of equations. Kumar *et al.* in [40] obtained exact solution of fully fuzzy linear system by solving a linear programming. In this paper, we propose Semi-iterative method for solving fully fuzzy linear systems.

This paper is organized as follows:

In Section 2 some basic definitions and arithmetic are reviewed. In Section 3 a new method is proposed for solving FFLS and we respectively give the semi-iterative method and some convenient iterative methods. In section 4 numerical results are considered to show the efficiency of the proposed method. Section 5 ends this paper with a conclusion.

2. Some Basic Definition and Arithmetic Operations

In this section, an appropriate brief introduction to preliminary topics such as fuzzy numbers and fuzzy calculus will be introduced and the definition for FFLS will be provided. For details, we refer to [26, 37].

Definition 2.1 Let X denote a universal set. Then a fuzzy subset $\widetilde{A}$ of X is defined by its membership function $\mu_{\widetilde{A}} : X \rightarrow [0,1]$; which assigns a real number $\mu_{\widetilde{A}}(x)$ in the interval [0,1], to each element $x \in X$, where the value of $\mu_{\widetilde{A}}(x)$ at x shows the grade of membership of x in $\widetilde{A}$.

A fuzzy subset $\widetilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\widetilde{A}}(x)$ and is often written $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)); x \in X\}$ .The class of fuzzy sets on X is denoted with $\Gamma(X)$.

Definition 2.2 A fuzzy set with the following membership function is named a triangular fuzzy number and in this paper we will use these fuzzy numbers.

$$\mu_{\widetilde{A}}(x) = \begin{cases} 1 - \dfrac{m-x}{\alpha}, & m - \alpha \le x \le m, \alpha > 0, \\ 1 - \dfrac{x-m}{\beta}, & m \le x \le m + \beta, \beta > 0, \\ 0, & \textit{else.} \end{cases}$$

Definition 2.3 A fuzzy number $\tilde{A}$ is said to be positive (negative) by $\tilde{A} > 0 (\tilde{A} < 0)$ if its membership function $\mu_{\tilde{A}}(x)$ satisfies $\mu_{\tilde{A}}(x) = 0, \forall x \le 0 (\forall x \ge 0)$.

Using its mean value and left and right spreads, and shape functions, such a fuzzy number $\tilde{A}$ is symbolically written $\tilde{A} = (m, \alpha, \beta)$. Obviously, $\tilde{A}$ is positive, if and only if $m - \alpha \ge 0$.

Definition 2.4 Two fuzzy numbers $\tilde{A} = (m, \alpha, \beta)$ and $\tilde{B} = (n, \gamma, \delta)$ are said to be equal, if and only if $m = n$, $\alpha = \gamma$ and $\beta = \delta$.

Definition 2.5 Let $\tilde{A} = (m, \alpha, \beta)$, $\tilde{B} = (n, \gamma, \delta)$ be two triangular fuzzy numbers then;

(i) $\tilde{A} \oplus \tilde{B} = (m, \alpha, \beta) \oplus (n, \gamma, \delta) = (m+n, \alpha+\gamma, \beta+\delta)$,

(ii) **)** $-\tilde{A} = -(m, \alpha, \beta) = (-m, \beta, \alpha)$,

(iii)if $\tilde{A}, \tilde{B}$ be a positive fuzzy number then: $(m, \alpha, \beta) \otimes (n, \gamma, \delta) \cong (mn, n\alpha + m\gamma, n\beta + m\delta)$,

(iv) For scalar multiplication we have;