**ISSN** 1746-7659, England, UK
**Journal of Information and Computing Science**
Vol. 9, No. 1, 2014, *pp*. 075-080

# The Hesitant Fuzzy Weighted OWA Operator and Its Application In Multiple Attribute Decision Making

Sun Min [1], Liu Jing [2]

[1] School of Mathematics and Statistics, Zaozhuang University, Shandong, 277160, China.
[2] School of Mathematics and Statistics, Zhejiang University of Finance and Economics, Hangzhou, 310018, China

(*Received June 20, 2013, accepted November 02, 2013*)

**Abstract.** The weighted ordered weighted averaging(WOWA) operator introduced by Torra is an important aggregation technique, which includes the famous weighted averaging(WA) operator and the ordered weighted averaging(OWA) operator as special cases. In this paper, we introduce a new hesitant fuzzy decision making technique called the hesitant fuzzy weighted OWA(HFWOWA) operator. It is an extension of the WOWA operator with the uncertain information represented as hesitant fuzzy numbers.It is shown that many existing hesitant fuzzy aggregation operators are the special cases of our proposed operator. We also study some of its main properties, such as commutativity, monotonicity and boundary.Moreover, an approach is proposed for multiple attribute decision making based on the proposed operator. Finally, an example is given to illustrate the developed method.

**Keywords:** hesitant fuzzy sets, multiple attribute decision making, HFWOWA operator.

## 1. Introduction

As an important extension of fuzzy set(FS), hesitant fuzzy set(HFS)[1,2] proposed by Torra and Narukawa, which allows the membership of an element to a set represent by several possible values, is a powerful tool to express people's hesitancy in real applications.

Since its appearance, the HFS has received more and more attention from many researchers in applied mathematics, computing science, management science and many others, and fruitful research works have been published about the HFS theory. For example, Xu and Xia[3] presented a lot of distance measures and similarity measures for HFSs. Based on the intuitionistic fuzzy entropy measures, Xu[4] proposed a variety of entropy measures for HFSs. Xia and Xu[5] developed a series of aggregation operators for hesitant fuzzy information, and discussed the relationship between the intuitionistic fuzzy set and the hesitant fuzzy set. Zhu et al.[6] developed some hesitant fuzzy geometric Bonferroni means to aggregate the hesitant fuzzy information.

On the other hand, the weighted averaging(WA) operator and the ordered weighted averaging(OWA) operator are two well-known aggregation techniques. The difference of the two operators is that the WA operator allows to weight each information source in relation to their reliability whereas the OWA operator allows to weight the values according to their ordering[7]. Therefore, weights represents different aspects in both the WA and OWA operators. However, both the operators consider only one of them. To solve this drawback, Torra[8] introduced the weighted ordered weighted averaging(WOWA) operator which can deal with the situation where both the importance of information sources and the importance of values have to be taken into account. Usually, when using the WOWA operator, it is assumed that the available information is clearly known and can be assessed with exact numbers. However, due to the increasing complexity of economic environment, it is very difficult for a decision maker to express his/her preferences over alternatives with exact numbers. In such cases, the fuzzy numbers may be more suitable. Therefore, in this paper, we will extend the famous WOWA operator to the hesitant fuzzy environment, and propose the hesitant fuzzy WOWA(HFWOWA) operator. It is a new hesitant fuzzy aggregation operator that uses the main characteristics of the WOWA operator and uncertain information represented in the form of hesitant fuzzy numbers. We also present a generalization of the HFWOWA operator by using the generalized mean. The generalization includes the HFWOWA operator and many other famous operators as special cases.

In order to do so, the remainder of this paper is organized as follows: In Section 2, we introduce some basic concepts related to HFSs and some well-known aggregation operators. In Section 3, we develop the hesitant fuzzy weighted OWA(IFWOWA) operator, and study its desirable properties and its special cases. We also give the two generalizations of the IFWOWA operator in this section. In Section 4, we shall apply the proposed operator to deal with MADM under hesitant fuzzy environments. In Section 5, an illustrative example is given to verify the proposed method, and some conclusions are given in the last section.

## 2. Preliminaries

This section briefly reviews the hesitant fuzzy set(HFS), the weighted averaging(WA) operator, the ordered weighted averaging(OWA) operator, the weighted ordered weighted averaging(WOWA) operator. Torra and Narukawa[1,2] gave the definition of HFS as follows.

**Definition 1** [1,2]. Let $X$ be a fixed set, and a hesitant fuzzy set(HFS) on $X$ is in term of a function that when applied to $X$ returns a subset of [0,1], which can be represented as the following mathematical symbol:

$$E = \{\langle x, h(x)\rangle \mid x \in X\} \tag{1}$$

where $h(x)$ is a set of some values in [0,1], denoting the possible membership degrees of the element $x \in X$ to the set $E$. For convenience, we call $h(x)$ a hesitant fuzzy element(HFE) and $H$ the set of all the HFEs.

**Definition 2** [5]. For a HFE $h$, $s(h) = \frac{1}{\sharp h}\sum_{\gamma\in h}\gamma$ is called the score function of $h$, where $\sharp h$ is the number of the elements in $h$. Moreover, for two HEFs $h_1$ and $h_2$, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Let $h, h_1$ and $h_2$ be three HFEs, then the operational laws on the HFEs are given as follows.

(1) $h^{\lambda} = \cup_{\gamma\in h}\{\gamma^{\lambda}\}$.

(2) $\lambda h = \cup_{\gamma\in h}\{1-(1-\gamma)^{\lambda}\}$.

(3) $h_1 \oplus h_2 = \cup_{\gamma_1\in h_1,\gamma_2\in h_2}\{\gamma_1+\gamma_2-\gamma_1\gamma_2\}$.

(4) $h_1 \otimes h_2 = \cup_{\gamma_1\in h_1,\gamma_2\in h_2}\{\gamma_1\gamma_2\}$.

The weighted averaging(WA) operator is a basic aggregation technique, which is defined as follows:

**Definition 3.** Let $a_i (i = 1, 2, \ldots, n)$ be a collection of real numbers. If

$$\mathrm{WA}(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} p_i a_i, \tag{2}$$

where $p = (p_1, p_2, \ldots, p_n)^{\bullet}$ is the weighting vector of $a_i (i = 1, 2, \ldots, n)$, such that $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0,1]$, then the mapping WA is called the weighted averaging(WA) operator.

Since its appearance, the ordered weighted averaging(OWA) operator, which was introduced by Yager[9] in 1988, has received more and more attention.

**Definition 4.** [9] An OWA operator of dimension $n$ is a mapping OWA: $R^n \to R$, defined by an associated weighting vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^{\bullet}$, such that $\sum_{i=1}^{n}\omega_i = 1$ and $\omega_i \in [0,1]$, according to the following formula:

$$\mathrm{OWA}(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} \omega_i a_{\sigma(i)}, \tag{3}$$