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A Fliter Method for Sloving LCP Based on Nonmonotone Line Search

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Abstract. In this paper, we propose a filter method to solve the linear complementarity problem(LCP). By using the Fisher-Burmeister function, we convert the LCP to an equivalent optimization problem with linear equality constraints. A filter approach is employed to tackle the optimization problem and the proposed mechanism for accepting the trial step is obtained by a nonmonotone filter technique. Under some conditions, we establish the global convergence of the algorithm.

Keywords: linear complementarity, Fisher-Burmeister function, constrained optimization, filter method, nonmontone technique, global convergence.

1. Introduction

In this paper, we consider the following linear complementarity problem (LCP)

$$\begin{gathered} y = Mx + q \\ x \geq 0, y \geq 0, x^T y = 0, \end{gathered} \tag{1}$$

where $M \in R^{n\times n}, x, y \in R^n$ and $x \geq 0 (y \geq 0)$ means that $x_i \geq 0 (y_i \geq 0)$ $i = 1, 2, \cdots, n$. In this paper, we assume that the solution set of (1) is nonempty, and let X denote the solution set of (1). For convenience, we use $w = (x^T, y^T)^T$.

LCP problem, arising in transportation, economy, engineering and many fields in the society, see [1,2] for survey. Optimization reformulation method is one of the most popular method for solving the LCP, one is the equivalent unconstrained optimization reformulation[3], and the other the equivalent constrained optimization reformulation[4]. In the last few years, a great deal of numerical methods had been proposed to deal with the responding optimization reformulation problems, such as nonsmooth Newton methods(see[4,5,6,7,8]), interior method(see[9])and smoothing method (see[10,11,12,13]and[14]for survey).

This paper will focus on the equivalent constrained optimization reformulation and the filter method to deal with linear equality constrained optimization reformulation of the LCP. The filter methods was proposed first by Fletcher and Leyffer[15], in which the use of a penalty function, a common feature of the large majority of the algorithms for constrained optimization, is replaced by the technique so-called "filter", and filter method has been actually applied in many optimization techniques, for instance, the pattern search method [16], the SLP method [17], the interior point method [18], the bundle approaches [19], the system of nonlinear equations and nonlinear least squares [20], multidimensional filter method[21], and so on.

In fact, filter method exhibits a certain degree of nonmonotonicity. The idea of nonmonotone technique can be traced back to Grippo et al.[22] in 1986. Due to its excellent numerical exhibition, over the last decades, the nonmonotone technique has been used in trust region method to deal with unconstrained and constrained optimization problems. Motivated by above ideas and methods, in this paper we use a filter algorithm that combines the nonmonotone technique for solving LCP.

The rest of paper is organized as follows: In the section 2, we state the knowledge summary and algorithm model. In the section 3 we analyze the convergence property of the algorithm. In the section 4, some discussions and remarks are given.

2. Knowledge summary and algorithm Model

It has been well known that by means of a suitable function: $\phi\ R^2 \to R$ the system

$$a \ge 0, b \ge 0, ab = 0, \tag{2}$$

can be transformed into an equivalent nonlinear equation

$$\phi(a,b) = 0, \tag{3}$$

In this situation, function ϕ is called as NCP-function. Then (1) can be reformulated as the following equivalent nonlinear equation system:

$$\Psi(x) = \begin{pmatrix} \phi(x_1, Mx+q)_1 \\ \vdots \\ \phi(x_n, Mx+q)_n \end{pmatrix}, \tag{4}$$

$$H(x,y) = \begin{pmatrix} \phi(x_1, y_1) \\ \vdots \\ \phi(x_n, y_n) \\ y - Mx - q \end{pmatrix}, \tag{5}$$

A lot of methods have been proposed to solve (4) or (5) to minimize their natural residual

$$\Phi_1(x) = \frac{1}{2}\|\Psi(x)\|^2 \quad or \quad \Phi_2(x) = \frac{1}{2}\|H(x,y)\|^2 \tag{6}$$

In general, (5) is nonsmooth and nonlinear, hence it is not easy to solve. However, in (5), the first *n* components are nonsmooth and nonlinear which is difficult to solve, contrarily to the first part, the last *n* components are easy to handle. Therefore, it is reasonable to handle the first part which consists of the *n* nonsmooth components and the second part which consists of the n linear equations separately. Based on this idea, we transform further (5) into the following equivalent minimization problem.

$$\min_{(x,y)\in R^{2n}} \Phi(x,y) = \frac{1}{2}\sum_{i=1}^{n} \phi\left(x_i, y_i\right)^2$$
$$s.t \qquad y - Mx - q = 0$$

Throughout the paper, we shall use the famous Fisher-Burmeister function defined by $\phi(a,b) = \sqrt{a^2+b^2} - a - b, (a,b \in R)$, which has many promised properties and attracted the attention of many researchers.

As mentioned in the former, we exploit the famous Fisher-Burmeister function. Then (1) can be converted to the equivalent nonlinear equation system (5).

$$\begin{aligned} &\min_{(x,y)\in R^{2n}} \Phi(x,y) = f(w) = \frac{1}{2}\sum_{i=1}^{n} \phi\left(x_i, y_i\right)^2 \\ &s.t \qquad y - Mx - q = 0, \end{aligned} \tag{7}$$

Since the algorithm we proposed in this paper converges to KKT point of (7). The first question needed to be answered is what condition guarantee that a KKT point of (1) is global solution of (7). Then, we easily know w^* solves (1) if and only if w^* solves (7). We have the following properties.

Lemma 2. 1[23] Function ϕ has the following properties:

(1) $\phi(a,b) = 0 \Leftrightarrow a \ge 0, b \ge 0, ab = 0$;