

Numerical solution of the nonlinear Fredholm-Volterra-Hammerstein integral equations via Bessel functions

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(Received July 10, 2013, accepted November 19, 2013)

Abstract. In this paper, a collocation method based on the Bessel polynomials are used for the solution of nonlinear Fredholm-Volterra-Hammerstein integral equations (FVHIEs). This method transforms the nonlinear (FVHIEs) in to matrix equations with the help of Bessel polynomials of the first kind and collocation points. The matrix equations corresponds to a system of nonlinear algebraic equations with the unknown Bessel coefficients. Present results demonstrate proposed method in comparison with other methods is more accurate, efficiency and reliability.

Keywords: Bessel polynomials, Integral equations, Collocation, Fredholm, Volterra, Hammerstein.

1. Introduction

In recent years, many different method have estimated the solution of integral equations. Also, for solution of these equations many analytical and numerical methods have been exited but most of the time numerical methods have been used to solve these equations. Ordokhani [1] has used Walsh-hybrid functions with Newton-Cotes nodes for solving of Fredholm-Hemmerstein integral equations. Authors [2] have solved nonlinear integral equations of Hammerstein type by Chebyshev polynomials. Maleknejad in [3], has used computational method based on Bernstein operational matrices for nonlinear Volterra-Fredholm-Hammerstein integral equations. Babolian and Shamsavaran in [4] have solved the nonlinear Fredholm integral equations of the second kind using Haar wavelets. Yousefi and Razzaghi in [5] have solved nonlinear Volterra-Fredholm integral equations by Legendre wavelers method. Yuzbasi et al. [6], Yuzbasi and Sezer [7], Yuzbasi et al. [8] have worked on the Bessel matrix and collocation methods for the numerical solutions of the neutral delay differential equations, the pantograph equations and the Lane-Emden differential equations. Also, readers who are interested to learn more about this topic could refer to [10 - 15].

Recently, Yazbasi in [16] used Bessel polynomials and Bessel collocation method [8] for solving high-order linear Fredholm-Volterra integro-differential equations. In this article, by Bessel polynomials and Bessel collocation method estimate solution of nonlinear (FVHIEs) to form:

$$y(x) = g(x) + \lambda_1 \int_a^b k_1(x,t)\psi_1(t,y(t))dt + \lambda_2 \int_a^x k_2(x,t)\psi_2(t,y(t))dt, \quad 0 \leq a \leq x, t \leq b, \quad (1)$$

where $y(x)$ is an unknown function, the known functions $g(x)$, $k_1(x,t)$, $k_2(x,t)$, $\psi_1(t,y(t))$ and $\psi_2(t,y(t))$. Also, λ_1 and λ_2 are real or complex constants.

2. Bessel polynomial of first kind

The m-th degree truncated Bessel polynomial of first kind are defined by [16]

$$J_m(x) = \sum_{k=0}^{\lfloor \frac{N-m}{2} \rfloor} \frac{(-1)^k}{k!(K+m)!} \left(\frac{x}{2}\right)^{2k+m}, \quad 0 \leq x < \infty, \quad m \in \mathbb{N}, \quad (2)$$

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where N is chosen positive integer so that $N \geq m$ and $m = 0, 1, \dots, N$. We are transform the Bessel polynomials of first kind to N -th degree Taylor basis functions. In matrix form as

$$J(x) = DX(x), \quad (3)$$

where

$$J(x) = [J_0(x), J_1(x), \dots, J_N(x)]^T, \quad X(x) = [1, x, x^2, \dots, x^N]^T. \quad (4)$$

If N is odd

$$D = \begin{bmatrix} \frac{1}{0!0!2^0} & 0 & \frac{-1}{1!1!2^2} & \cdots & \frac{(-1)^{\frac{N-1}{2}}}{(\frac{N-1}{2})!(\frac{N-1}{2})!2^{N-1}} & 0 \\ 0 & \frac{1}{0!1!2^1} & 0 & \cdots & 0 & \frac{(-1)^{\frac{N-1}{2}}}{(\frac{N-1}{2})!(\frac{N-1}{2})!2^N} \\ 0 & 0 & \frac{1}{0!2!2^2} & \cdots & \frac{(-1)^{\frac{N-3}{2}}}{(\frac{N-3}{2})!(\frac{N+1}{2})!2^{N-1}} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{0!(N-1)!2^{N-1}} & 0 \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{0!N!2^N} \end{bmatrix}_{(N+1) \times (N+1)}.$$

If N is even

$$D = \begin{bmatrix} \frac{1}{0!0!2^0} & 0 & \frac{-1}{1!1!2^2} & \cdots & 0 & \frac{(-1)^{\frac{N}{2}}}{(\frac{N}{2})!(\frac{N}{2})!2^N} \\ 0 & \frac{1}{0!1!2^1} & 0 & \cdots & \frac{(-1)^{\frac{N-2}{2}}}{(\frac{N-2}{2})!(\frac{N}{2})!2^{N-1}} & 0 \\ 0 & 0 & \frac{1}{0!2!2^2} & \cdots & 0 & \frac{(-1)^{\frac{N-2}{2}}}{(\frac{N-2}{2})!(\frac{N+2}{2})!2^N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{0!(N-1)!2^{N-1}} & 0 \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{0!N!2^N} \end{bmatrix}_{(N+1) \times (N+1)}.$$

3. Fundamental relations

3.1. Matrix relation for the Fredholm integral part

In this section we can approximate the kernel function $k_1(x, t)$ by the truncated Maclaurin series and truncated Bessel series [16], respectively

$$k_1(x, t) = \sum_{m=0}^N \sum_{n=0}^N {}_t k_{mn}^1 x^m t^n, \quad (5)$$

$$k_1(x, t) = \sum_{m=0}^N \sum_{n=0}^N {}_b k_{mn}^1 J_m(x) J_n(t),$$

Where