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Solving Fractional Nonlinear Fredholm Integro-differential Equations via Hybrid of Rationalized Haar Functions

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Abstract. This paper presents hybrid of rationalized Haar (HRH) functions method for approximate the numerical solution of the fractional nonlinear Fredholm integro-differential equations (FNFIDEs). The fractional derivatives are considered in the sense of Caputo. The fractional operational matrix of hybrid of block-pulse and rationalized Haar functions are presented. This matrix together with the dual operational matrix are used to reduce the computation of FNFIDEs into a system of algebraic equations. Some numerical examples are given and the results of applying this method demonstrate time and computational are small.

Keywords: Fredholm integro-differential equation, Riemann-Liouville integral, Caputo fractional derivative, Fractional operational matrix, Rationalized Haar, Hybrid.

1. Introduction

The main purpose of this paper is to consider the numerical solution of the FNFIDEs of the types

$$y'(x)-\lambda\int_0^1 k(x,t)_*D^{\alpha}y(t)dt = f(x), \tag{1}$$

$$0\le x\le 1,\quad 0<\alpha<1,$$

with the initial condition

$$y(0)=\gamma,$$

and

Error! Not a valid embedded object.

$$_*D^{\alpha}y(x)-\lambda\int_0^1 k(x,t)G(y(t))dt = f(x), \tag{2}$$

$$0\le x\le 1,\quad n-1<\alpha\le,\quad n\in N,$$

$y^{(i)}(0)=\delta_i$, with n initial conditions

$i=0,1,2,3,\ldots,n-1.$

Here $_*D^{\alpha}$ is Caputo's fractional derivative, α is a parameter describing the order of fractional derivative and λ is a real known constant. Also, $f\in L^2([0,1])$ and $k\in L^2([0,1]^2)$ are given functions, y(x) is the solution to be determined and G(y(x)) is a polynomial of the unknown function y(x), we assume $G(y(x))=[y(x)]^q$ where $1<q\in N$. The fractional calculus has been applied in many mathematica models. For example the nonlinear oscillation of earthquake [1], fluid-dynamic traffic [2], continuum and statistical mechanics [3] can be modeled with fractional derivatives. There are several methods that are used to solve the fractional integro-differential equations such as, Adomian decomposition method [4], collocation method [5], CAS wavelet method [6], hybrid functions and the collocation method [7] and second kind Chebyshev

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wavelet method [8]. In this work we report application of HRH functions to solve the FNFIDEs, for this purpose, in problems (1) and (2) we expanding the high order of derivative by HRH functions with unknown coefficients, then we can evaluate the unknown coefficients and obtain an approximate solution to problems (1) and (2). In this technic time and computational are small and this is a good and useful property of the HRH functions method.
The article is organized as follows:

In section 2, we introduce some necessary fundamentals of the fractional calculus theory. in section 3, we present the properties of HRH functions required for our subsequent development. In section, 4 we describe the solution of problems (1) and (2) by using HRH functions, and in section 5 we give some numerical examples to demonstrate the accuracy of the proposed method.

2. Fundamentals of fractional calculus

In this section, we give some definitions and fundamentals of the fractional calculus theory.
Definition 2.1. The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$ is defined as [9,10]

$$I^{\alpha} y(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} y(t)dt, \quad \alpha > 0, \quad x > 0,$$

$$I^0 y(x) = y(x),$$

whrere $\Gamma(.)$ is Gamma function.
It has the following properties:

$$I^{\alpha} y(x) = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}, \quad \gamma > -1.$$

Definition 2.2. The Caputo definition of fractional derivative operator is given by [11,12]

$$_*D^{\alpha} y(x) = I^{n-\alpha}\, _*D^{\alpha} y(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} y^{(n)}(t)dt,$$

where $n-1 \leq \alpha < n, \quad n \in N, \quad \alpha > 0.$
It has following properties:

$$_*D^{\alpha} I^{\alpha} y(x) = y(x),$$

$$I^{\alpha}\, _*D^{\alpha} y(x) = y(x) - \sum_0^{n-1} y^{(k)}(0^+) \frac{x^k}{k!}, \quad x > 0.$$

3. Properties of hybrid functions

3.1 Hybrid functions of block-pulse and rationalized Haar functions

The HRH functions $h_{nr}(x)$, $n = 1,2,...,N$, $R = 0,1,2,...,M-1$, $M = 2^{\beta+1}$, $\beta = 0,1,2,...,$ where n, r are the order of block-pulse functions and rationalized Haar functions respectively is defined on the interval [0,1) as [13]

$$h_{nr}(x) = \begin{cases} h_{nr}(Nx-n+1), & \frac{n-1}{N} \leq x < \frac{n}{N}, \\ 0, & otherwise, \end{cases} \tag{3}$$