

Internet Traffic Modelling -Variance Based Markovian Fitting of Fractal Point Process from Self-Similarity Perspective

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Abstract. Most of the proposed self-similar traffic models could not address fractal onset time at which self-similar behavior actually begins. This parameter has considerable impact on network performance. Fractal point process (FPP) emulates self-similar traffic and involves fractal onset time (FOT). However, this process is asymptotic in nature and has less effective in queueing based performance. In this paper, we propose a model of variance based Markovian fitting. The proposed method is to match the variance of FPP and superposed Markov modulated Poisson Process (MMPP) while taking FOT into consideration. Superposition consists of several interrupted Poisson processes (IPPs) and Poisson process. We present how well resultant MMPP could approximate FPP which emulates self-similar traffic. We investigate queueing behavior of resultant queueing system in terms of a packet loss probability. We demonstrate how FOT affects the fitting model and queueing behavior. We conclude from the numerical example that network nodes with a self-similar input traffic can be well represented by a queueing system with MMPP input

Keywords: *Self-similarity; Fractal point process; fractal onset time; Markov modulated Poisson process, Variance; Packet loss probability.*

1. Introduction

Seminal studies revealed the presence of self-similarity or long range dependence (LRD) in LAN, WAN, the variable bit rate (VBR) video traffic, and its impact on the network traffic [1-3]. This type of traffic exhibits statistical similarity over different time scales and is highly correlated. Characterizing the statistical behavior of traffic is crucial to proper design of routers to provide the quality of service (QoS). If the traffic models do not accurately represent the real traffic, then the network performance may be estimated over or underestimated [4]. Traffic models such as Fractional Brownian Motion (FBM), Fractional Auto Regressive Integrated Moving Average (FARIMA), Chaotic maps are proposed to characterize the self-similarity. Although, these processes are parsimonious, but are less effective in the case of queueing based performance evaluation when buffer sizes are small. In [5-8], Markovian arrival process (MAP) is employed to model self-similar behavior over the different time scales. These fitting models equate the second order statistics of self-similar traffic and that of superposition of several 2-state Markov modulated Poisson Process (MMPP) over desired time-scales. However, in the paper [5], covariance function of resultant MMPP is approximated by suppressing the higher order terms in Taylor's expansion. In the paper [6], MMPP emulating the self-similar traffic is fitted by matching variance over the desired time-scales. Resultant MMPP here is superposition of several Interrupted Poisson Process (IPPs) wherein two modulating parameters of each IPP are equal. The fitting method [6, 7] is generalized in the paper [8] by taking distinct modulating parameters in each IPP. Paulo Salvador et.al [9] proposed a model to fit discrete time MMPP that matches both autocovariance and marginal distribution of the counting process in such a way that model can capture self-similar behavior up to the time-scales of interest. Fractal onset time (FOT) defines the time scale from which self-similar behavior begins and is denoted by T_0 [11]. In the paper [12], the impact of FOT is realized besides the impact of another important characteristic Hurst parameter H of the self-similar traffic. According to the measurement studies, FOTs of the network traffic are at scales in the order of a few hundreds of milliseconds. The FOT plays an important role in characterizing the burstiness of the network traffic. In the said papers, the Markov-modulated Poisson process (MMPP) emulating the self-similar traffic over the different time scale is fitted, however, the time scale where self-similar nature actually begins is not considered.

Fractal point processes (FPPs) are proved to be self-similar [12], and they provide network traffic models [13]. The second order statistics of FPP involve not only the Hurst parameter but also FOT [12-13].

However, these processes are asymptotic in nature and has less effective in queueing based performance when the buffer sizes are small. That is, FPP can be used as a self-similar traffic generator, but it is not so useful in the context of the queueing theory. Hence, in this paper, first, we fit the MMPP for FPP by equating the variance while taking FOT into consideration and then we model network nodes such as routers by the $MMPP/D/1/K$ queueing system to investigate the queueing behavior. It is found from the numerical results that the MMPP model could emulate both the FPP traffic and the exact self-similar traffic. There are two objectives with the fitting described in this paper. First one is that, the resulting MMPP which works well for the queueing theory will have same statistical characteristics as that of FPP and self-similar process. The second one is to investigate queueing behavior under any traffic conditions. For the first objective, variance-time results of the self-similar traffic, FPP, and resultant MMPP are presented. In the context of the second objective, packet loss probability (PLP) against traffic intensity is presented.

The rest of the paper is organized as follows. In section 2, we first overview the definitions of self-similar process and FPP. In section 3, we present the fitting procedure. We then present the analytical results of $MMPP/D/1/K$, in section 4. In section 5, we demonstrate accuracy of the proposed model by means of numerical results. Finally, some conclusions are made in section 6.

2. Self-Similar Process and Fractal Point Process (FPP)

The second order statistics, namely variance, index of dispersion of counts (IDC), and auto covariance function (ACF) are relatively straightforward to fit the parameters of a model emulating self-similar traffic and gives much information [11]. As a result, these statistics are exploited by several authors. In this section, first we overview the definition of the self-similar process and the fractal point processes in terms of the second order statistics.

2.1 Self-Similar Process

Consider X to be a second -order stationary process with variance σ^2 , and divide time axis into disjoint intervals of unit length, we could define $X = \{X_t / t=1,2,3,\dots\}$ to be a number of points (packet arrivals) in the t^{th} interval. A new sequence $X^{(m)} = X_t^{(m)}$, where $X_t^{(m)} = \frac{1}{m} \sum_{i=1}^m X_{(t-1)m+i}$, $t = 1, 2, 3, \dots$

is the average of the original sequence in m non-overlapping blocks. Then the definitions of the exact second-order self-similar process and the asymptotically second-order self-similar process are defined as follows:

Definition 1. The second-order stationary process X is defined as an exact second-order self-similar process with the Hurst parameter, $H = 1 - \beta/2$ if

$$\text{Var}(X^{(m)}) = \sigma^2 m^{-\beta}, \forall m \geq 1.$$

Definition 2. The second-order stationary process X is defined as an exact second-order self-similar process with the Hurst parameter, $H = 1 - \beta/2$ if

$$r(k) = \frac{1}{2} \nabla^2(k^{2-\beta})$$

where $r(k)$ is an auto covariance function, $\nabla^2(.)$ is the second central difference operator and it is defined as

$$\nabla^2(f(k)) = f(k+1) - 2f(k) + f(k-1).$$

Definition 3. The process X is called asymptotically second-order self-similar process with the Hurst parameter, $H = 1 - \beta/2$ if

$$r^{(m)}(k) \rightarrow \frac{1}{2} \nabla^2(k^{2-\beta}) \quad \text{as } m \rightarrow \infty.$$