

A single-input linear controller for complete synchronization of a delay financial hyperchaotic system

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Abstract. This paper is involved with the complete synchronization problems for two identical delay financial hyperchaotic system with different initial conditions, and a simple complete synchronization scheme only with a single linear input is proposed. Based on the Lyapunov stability theory, both linear feedback control and adaptive control approaches is derived to complete synchronization between two nearly identical delay financial hyperchaotic systems with unknown parameters is also studied. Numerical simulation results are showing the effectiveness of the proposed hyperchaotic synchronization method.

Keywords: delay financial hyperchaotic system, complete synchronization, linear feedback control, adaptive control.

1. Introduction

In recent years, chaos study has increasingly become an important topic in nonlinear areas. And chaotic synchronization has been developed extensively in the last few years. Since Pecora and Carrol proposed a successful method to synchronize two identical chaotic systems with different initial conditions [1]. Chaos synchronization has been widely explored and studied because of its potential applications in secure communication, chemical reactions, biological systems, information science, plasma technologies, etc. Meanwhile, many synchronization schemes have been proposed [2-7] such as complete synchronization, phase synchronization, anti-synchronization, lag synchronization, generalized synchronization and projective synchronization and so on.

Economic dynamics has recently become more prominent in mainstream economics [8, 9]. However, with the development of economy, the old financial chaotic system cannot meet the needs of the market. Therefore, more and more scholars improve it by adding an additional state variable [10-13]. Recently, a novel financial hyperchaotic system was brought up [14]. The dynamical behaviors of the new system are more complex, and effective controls are implemented. But there are barely any studies on its synchronization which is the main job we did in this paper. And in a practical way, a smaller number of controllers and simpler form of controllers are greatly practical. The linear feedback control technique was used to synchronize chaotic system[15,16] in various research works.

In this paper, we will adapt the single-input linear feedback controller to investigate the synchronization between these two identical time-delay financial hyperchaotic systems. Base on the Lyapunov stability theory and the adaptive control theory, the single-input adaptive controller associates with estimated update laws to synchronize two nearly identical delay hyperchaotic systems with unknown parameters.

We organize our paper as follows. In Section 2, the financial hyperchaotic system with time delay is present. In Section 3, Synchronization between two identical new delay hyperchaotic systems via single-input linear feedback control and adaptive feedback control laws are proposed and show the synchronization with unknown parameters in the response system. The numerical simulations are also presented. Finally, the conclusions are drawn in Section 4.

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2. The financial hyperchaotic system with time delay

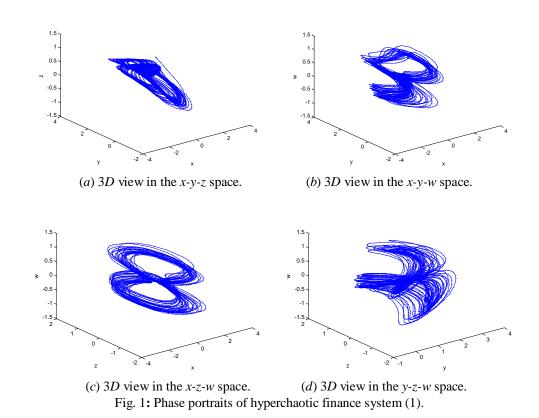
The novel financial hyperchaotic system is a non-delay financial hyperchaotic system in paper [14]. Based on the novel financial hyperchaotic system in this paper, we put forward a delay financial hyperchaotic system by plus a time delay on the average profit margin w in the first equation.

The model describes the time variations of four state variables: the interest rate x, the investment demand y, the price exponent z, and the average profit margin w.

The financial hyperchaotic system with time delay is described as

$$\begin{cases} \dot{x}_{1} = z_{1} + (y_{1} - a)x_{1} + w_{1}(t - \tau), \\ \dot{y}_{1} = 1 - by_{1} - x_{1}^{2}, \\ \dot{z}_{1} = -x_{1} - cz_{1}, \\ \dot{w}_{1} = -dx_{1}y_{1} - kw_{1}, \end{cases}$$
(1)

where $\tau > 0$ is the time delay, a, b, c, d, k are the parameters of the system (1), and they are positive constants. When $\tau = 0$, system (1) is the financial hyperchaotic system [14]. For convenience, we call it delay financial hyperchaotic system. When the parameters are a = 0.9, b = 0.2, c = 1.5, d = 0.2, and k = 0.17, the four Lyapunov exponents of the system (4) calculated with Wolf algorithm are $L_1 = 0.034432$, $L_2 = 0.018041$, $L_3 = 0$, and $L_4 = -1.1499$. Figure 1(a)–(d) show the 3-dimensional phase portraits of financial hyperchaotic system (4).



3. Complete synchronization of delay financial hyperchaotic system

In this subsection, we will investigate the synchronization of two identical delay financial hyperchaotic system via a single feedback control only with one variable. For this purpose, the drive hyperchaotic time-delay system is chosen as (1), and the response system is given as follows: