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A steepest descent method with a Wolf type line search

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Abstract. The steepest decent method is proposed by French mathematician Cauchy and it is one of the simplest and oldest methods for solving unconstrained optimization problems. The steepest descent method, negative gradient direction is chosen as the search direction, also known as the gradient method. Usually, the step length α_k of the steepest decent method can be computed by some inexact line search. In this paper, we will use a Wolfe type line search to evaluate the step length and its convergence property will be given under mild assumptions. From the numerical results, we can see that the steepest descent method with this Wolfe type line search is very promising. Finally, we give an application of the method to solve the nonlinear complementarity problem.

Keywords: the steepest decent method, line search, global convergence

1. Introduction

We consider the following unconstrained optimization problem

$$\min_{x\in R^n} f(x), \tag{1}$$

where $f: R^n \to R$ is continuously differentiable. In the past four decades, many theories and algorithms in globally optimal problems have been developed ([1-15]). Among these methods, the steepest decent method proposed by French mathematician Cauchy is one of the well-known and practical methods for global optimization problems. Because it has the simple structure, the less amount of calculation and the fast convergence speed when it away from the minimum value of the problem. So the steepest descent method has become one of the most famous methods for solving large-scale optimization problem. The steepest descent method has been studied deeply by many scholars, such as [1-4]. In practice, it often used in communication, computer and information engineering and other fields. Therefore, it has an important meaning for the study of the steepest descent method.

The classical steepest descent method for the continuously differentiable function $f: R^n \to R$ is defined by the iteration

$$x_{k+1} = x_k + \alpha_k d_k,$$

where d_k is the steepest descent direction at x_k and α_k is the step length at x_k .Since we use a negative gradient direction as the search direction, so the steepest decent method is also sometimes called the gradient method. In practice, the step length is computed by an inexact line search, this ensures an appropriate reduction in the objective function. Usually, we use Armijo, Wolfe and other inexact line search rule to compute the step length α_k (such as [2]). In this paper, we use a Wolfe type line search to computing the step length. The numerical results in Section 4 indicate that, for some problems, the steepest descent method with Wolfe type line search can obtain the global minimum solutions most close to the function, as well as the steepest descent with Armijo line search.

The Wolfe type line search (see [5]) as follows:

Choose $\alpha_k > 0$ such as

$$f(x_k) - f(x_k + \alpha_k d_k) \geq \rho\alpha_k^2 \|d_k\|^2 \tag{2}$$

$$g(x_k + \alpha_k d_k)^T d_k \geq -2\sigma\alpha_k \|d_k\|^2 \tag{3}$$

where $0 < \rho < \sigma < 1$.

This paper is organized as follows. In Section 2, we will give the algorithm of the steepest descent method with the Wolfe type line search and the steepest descent method with Armijo line search, respectively. The global convergence of the methods will be given in Section 3. In Section 4, some numerical results are reported for both the steepest descent method with the Wolfe line search, the steepest descent method with Armijo line search and some conjugate gradient methods. Finally, in Section 5, we give the conclusion and an application of the steepest descent method for solving nonlinear complementarity problem.

In this paper, we denote $g_k = g(x_k) = \nabla f(x_k)$. The norm $\|\cdot\|$ is the Euclidean norm.

2. Algorithm

In this section, we will give the steepest descent method with the Wolfe type line search and the Armijo line search, respectively. The steepest descent method with Armijo line search has been detail introduced in [2], so we will only give the algorithm.

Algorithm 2.1

Step 0. Given $x_0 \in R^n$, $\beta \in (0,1)$, $\sigma \in (0,0.5)$, $0 < \varepsilon << 1$, $k = 1$.

Step 1. Compute g_k . If $\|g_k\| \le \varepsilon$, stop. Output x_k as the approximate optimal solution.

Step 2. Choose $d_k = -g_k$.

Step 3. Compute α_k by

$$f(x_k + \beta^m d_k) \le f(x_k) + \sigma\beta^m g_k^T d_k .$$

Step 4. Set $x_{k+1} = x_k + \alpha_k d_k$, $k = k+1$, go to Step 1.

In the following, we present the steepest descent method with the Wolfe type line search.

Algorithm 2.2

Step 0. Given $x_0 \in R^n$, $\beta \in (0,1)$, $\sigma \in (0,0.5)$, $0 < \varepsilon << 1$, $k = 1$.

Step 1. Compute g_k . If $\|g_k\| \le \varepsilon$, stop. Output x_k as the approximate optimal solution.

Step 2. Choose $d_k = -g_k$.

Step 3. Compute α_k by (2) and (3).

Step 4. Set $x_{k+1} = x_k + \alpha_k d_k$, $k = k+1$, go to Step 1.

As a comparison, in Section 4, we will give the numerical results of the above two algorithms, respectively.

3. Convergence analysis of the algorithms

The convergence result of Algorithm 2.1 has been given in [2] in detail. In the following, we only give the global convergence result of Algorithm2.2.

In order to establish the global convergence of Algorithm 2.2, firstly, we give the following Assumption **3.1 and Lemma 3.1.**

Assumption 3.1. $f(x)$ is bounded below on the level set

$$L_0 = \left\{ x \in R^n \,\middle|\, f(x) \le f(x_0) \right\}.$$

Lemma 3.1. Suppose that Assumption 3.1 holds, then the Wolfe type line search (2) and (3) is feasible (see [5]).

Now, we give the following global convergence theorem for the steepest descent method with the Wolfe type line search under mild assumptions.