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Bounded Extended Cesàro Operators From Q_K Spaces into Weighted Bloch Spaces

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Abstract. Sufficient and necessary conditions for extended Cesàro operators from Q_K spaces into weighted Bloch spaces B_μ and logarithmic Bloch spaces $B_{\log}$ in the unit disc to be bounded are obtained.

Keywords: Cesàro operators, Q_K spaces, weighted Bloch spaces, logarithmic Bloch spaces

1. Introduction

Let D be the open unit disc of the complex plane C, $H(D)$ be the space of all analytic functions in D. A positive continuous decreasing function on the interval $[0,1)$ is called a normal function if there are constants a, b, δ such that $0<\delta<1$, $0<a<b<+\infty$, and $\frac{\mu(r)}{(1-r)^a}$ is decreasing and $\frac{\mu(r)}{(1-r)^b}$ is increasing on $[\delta,1)$,

Moreover, $\lim_{r\to 1^-}\frac{\mu(r)}{(1-r)^a}=0$ and $\lim_{r\to 1^-}\frac{\mu(r)}{(1-r)^b}=+\infty$. For $z\in D$, we can extend its definition, $\mu(z)=\mu(|z|)$.

The weighted Bloch spaces

$$B_\mu=\left\{f\in H(D)\,\middle|\,\|f\|_{B_\mu}=\sup_D \mu(z)|f'(z)|<+\infty\right\}$$

are Banach spaces under the norms $\|f\|_{B_\mu}=|f(0)|+\sup_D\mu(z)|f'(z)|$. Specially, when $\mu(z)=\left(1-|z|^2\right)^\alpha$, $0<\alpha<+\infty$, we get α-Bloch spaces B_α; when $\mu(z)=\left(1-|z|^2\right)\log\left(2/\left(1-|z|^2\right)\right)$, we get logarithmic Bloch space $B_{\log}$.

Let $Aut(D)$ be the holomorphic automorphism group on D under composite transformations of D. For $a\in D$, $\phi_a(z)=(z-a)/(1-\bar{a}z)\in Aut(D)$, green function $g(z,a)$ on D with pole $a\in D$ is given by

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$g(z,a)=\log\frac{1}{|\phi_a(z)|}$, $dA=\frac{dxdy}{\pi}$ is the normalized Lebesgue area measure, the Banach spaces Q_K spaces consist of those $f\in H(D)$ such that

$$\|f\|_{Q_K}^2=\sup_D\int_D|f'(z)|^2K(g(z,a))dA<+\infty,$$

where $K:(0,+\infty)\to[0,+\infty)$ is right continuous nondecreasing function, $\varphi_K(s)=\sup_{0\le t\le 1}K(st)/K(t)$, $0<s<+\infty$, hence φ_K also is right continuous nondecreasing function. Suppose that φ_K always satisfies the following conditions: (for more dedails, please see [1] [2])

$$\int_0^1\varphi_K(s)\frac{ds}{s}<+\infty, \qquad \int_1^{+\infty}\varphi_K(s)\frac{ds}{s^2}<+\infty. \tag{1}$$

For a holomorphic function $f\in H(D)$ with Taylor expansion $f(z)=\sum_{n=0}^{+\infty}a_nz^n$, Cesàro operator C acting on f is defined by

$$C[f](z)=\sum_{n=0}^{+\infty}\left(\frac{1}{n+1}\sum_{k=0}^{n}a_k\right)z^n.$$

By computation, we see that

$$C[f](z)=\frac{1}{z}\int_0^z f(\zeta)\frac{1}{1-\zeta}d\zeta=\frac{1}{z}\int_0^z f(\zeta)\left(\log\frac{1}{1-\zeta}\right)'d\zeta.$$

On most holomorphic function spaces, $C[f]$ is bounded if and only if the integral operator $f\to\int_0^z f(\zeta)\left(\log\frac{1}{1-\zeta}\right)'d\zeta$ is bounded. From this point of view, it's natural to consider the extended Cesàro operator T_g with holomorphic symbol $g\in H(D)$:

$$(T_gf)(z)=\int_0^z f(\zeta)g'(\zeta)d\zeta.$$

Sufficient and necessary conditions for the Cesàro operator on Q_K space in the unit disc to be bounded were given in [3], boundedness of the Cesàro operator on α-Bloch spaces B_α was studied in [4][5], Sufficient and necessary conditions for the extended Cesàro operator T_g from Q_K space into α-Bloch spaces B_α to be bounded were obtained in [6]. However, in this paper, we generalize the results in [6], characterise boundedness of the extended Cesàro operator T_g from Q_K space into the weighted Bloch spaces B_μ in the unit disc, and discuss some relationships between bounded extended Cesàro operators.

2. Bounded extended Cesàro operators

Lemma 1 [2] If K satisfies the condition (1), then we have $\log(1-z)\in Q_K$.