

Modified simple equation method to the nonlinear Hirota Satsuma KdV system

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Abstract. In this article, we use the modified simple equation method to construct the exact traveling wave solutions for some nonlinear PDE's in mathematical physics namely the coupled Hirota – Satsuma KdV equations and the generalized coupled Hirota – Satsuma KdV equations. Based on this formulation, solitary solutions can be easily obtained by using the proposed method. Some known solutions obtained by the tanh-coth method are recovered as special cases. The proposed method is direct and more powerful than the other methods.

Keywords: The modified simple equation method, Exact solutions, Nonlinear evaluation equations, The nonlinear Hirota Satsuma equations.

1. Introduction

The nonlinear complex physical phenomena are related to nonlinear partial differential equations (NLPDEs) which are involved in many fields from physics to engineering, biology, chemistry, mechanics, etc. The exact solutions to nonlinear partial differential equations (NLPDEs) play an essential role in the nonlinear science, in that they may provide much physical information and help one to understand the mechanism that governs these physical models. Many powerful methods have been presented to construct the each solutions such as the Backlund transform [1], the homogeneous balance method [2], the extended tanh-function method [3,4], the F-expansion method [5], the exp-function expansion method [6], the generalized Riccati equation [7,8], the sub-ODE method [9,10], the extended sine-cosine method [11], the complex hyperbolic function method [12,13], the (G'/G) -expansion method [14,15,16], the modified rational Jacobi elliptic functions method [17], and so on.

In this article, we use the modified simple equation method [18,19,20,21,22] to calculate the exact solutions of some NLPDEs in mathematical physics, namely the coupled Hirota – Satsuma KdV equations and the generalized coupled Hirota – Satsuma KdV equations.

2. Summary of the modified simple equation method

In this section, we would like to outline the main steps of this method as follows:

We consider the following nonlinear partial differential equation

$$U(u, u_x, u_t, u_{xt}, u_{xx}, u_{tt}, \dots) = 0. \quad (2.1)$$

Step 1. We use the following travelling wave transformation

$$u(x, t) = u(\xi), \quad \xi = x + ct, \quad (2.2)$$

where c is an arbitrary constant to be determined latter. The transformation (2.2) permits us to convert the PDE (2.1) to the ODE in the following form

$$P(u, u', u'', \dots) = 0, \quad (2.3)$$

where P is a polynomial in u and its total derivatives with respect ξ .

Step 2. We suppose that equation (2.3) has the following solution form

$$u(\xi) = \sum_{k=0}^N A_k \left[\frac{\psi'(\xi)}{\psi(\xi)} \right]^k, \quad (2.4)$$

where A_k are arbitrary constants to be determined later, such that $A_N \neq 0$, and $\psi(\xi)$ is an unknown function to be determined later.

Step 3. We determine the positive integer N of the formal polynomial solution equation (2.4) by balancing the highest nonlinear terms and the highest-order derivatives in equation (2.3).

Step 4. We substitute (2.4) into (2.3); we calculate all the necessary derivatives $u', u'', \dots$, and then we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\frac{\psi'(\xi)}{\psi(\xi)}$ and its derivatives. In this polynomial, we equate all the coefficients of it with zero. This operation yields a system of equations which can be solved to find A_k and $\psi(\xi)$. Consequently, we can get the exact solution of Eq. (2.1).

3. Applications

The AODV is a reactive routing protocol that combines the advantages of both protocols, Dynamic Source Routing (DSR) and DSDV [22]. If a node using AODV protocol for communication, send a message to a destination node for which it does not have a valid route to, it initiates a route discovery process to locate the destination node [23].

In this section, we use the modified simple equation method to construct the exact solutions for the coupled Hirota – Satsuma KdV equations and the generalized coupled Hirota – Satsuma KdV equations which are very important in the mathematical physics and have been bailed attention by many researcher in physics and engineering.

3.1. Example 1. The coupled Hirota – Satsuma KdV equations

In this section, we study the coupled Hirota – Satsuma KdV equations[23,24]

$$\begin{aligned} u_t &= \frac{1}{4}u_{xxx} + 3uu_x - 6vv_x, \\ v_t &= -\frac{1}{2}v_{xxx} - 3uv_x. \end{aligned} \quad (3.1)$$

We use the travelling wave transformation,

$$u = U(\xi), \quad v = V(\xi), \quad (3.2)$$

where $\xi = x - \lambda t$. The transformation (3.2) permit us to convert Eqs.(3.1) into the following ordinary differential equations

$$\begin{aligned} \lambda U + \frac{1}{4}U'' + \frac{3}{2}U^2 - 3V^2 + C_1 &= 0, \\ -\lambda V' + \frac{1}{2}V'' + 3UV' &= 0, \end{aligned} \quad (3.3)$$

where C_1 is the integration constant. Balancing the highest order derivative U'' and the nonlinear term U^2 and V^2 in Eq.(3.3), we get $N = M = 2$. Consequently, the solutions of Eq. (3.3) takes the following form

$$\begin{aligned} U &= a_0 + a_1 \left(\frac{\psi'(\xi)}{\psi(\xi)} \right) + a_2 \left(\frac{\psi'(\xi)}{\psi(\xi)} \right)^2, \\ V &= b_0 + b_1 \left(\frac{\psi'(\xi)}{\psi(\xi)} \right) + b_2 \left(\frac{\psi'(\xi)}{\psi(\xi)} \right)^2, \end{aligned} \quad (3.4)$$

where a_0, b_0, a_1, b_1, b_2 , are constants to be determined later, and $\psi(\xi)$ is an arbitrary function to be determined later. Substituting (3.4) into (3.3) and equating all the coefficients of $\psi(\xi)$ to be zero, we obtain