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Application of homotopy perturbation and Adomian decomposition methods for solving an inverse heat problem

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Abstract. In this paper, the homotopy perturbation method is proposed to solve an inverse problem of finding an unknown function in parabolic equation with overspecified data. Comparison is made between Adomian decomposition method and the proposed method. It is shown; Adomian decomposition method is equivalent to the homotopy perturbation method in the model problem. To show the efficiency of these methods, several test problems are presented for one-, two- and three-dimensional cases. Comparison of the applied methods with exact solutions reveals that both methods are tremendously effective.

Keywords: Homotopy perturbation method (HPM), Adomian decomposition method (ADM), inverse parabolic problem, integral overspecified data.

1. Introduction

In this article, we consider the following inverse problem. Find $u(\boldsymbol{x},t)$ and $p(t)$ satisfy:

$$u_t(\boldsymbol{x},t) = \Delta u(\boldsymbol{x},t) + p(t)u(\boldsymbol{x},t) + g(\boldsymbol{x},t); \qquad o < t < T, \qquad \boldsymbol{x} \in \Omega, \tag{1}$$

with initial condition:

$$u(\boldsymbol{x},0) = u_0(\boldsymbol{x}) \qquad \boldsymbol{x} \in \Omega, \tag{2}$$

and boundary conditions:

$$u(\boldsymbol{x},t) = \mathrm{h}(\boldsymbol{x},t); \quad o < t < T, \qquad \boldsymbol{x} \in \partial\Omega. \tag{3}$$

An additional boundary condition which can be the integral overspecification over the spatial domain for (1) is given in the following form:

$$\int_{\Omega} u(\boldsymbol{x},t)d\boldsymbol{x} = E(t); \qquad o < t < T, \tag{4}$$

where Δ is the Laplace operator, $\Omega = [0,1]^d$ is spatial domain of the problem for $d = 1,2,3$, $\partial\Omega$ is the boundary of Ω, $\boldsymbol{x} = (x_1, \dots, x_d)$ and g, u_0, h and E are known functions, while u and p are unknown. The integral overspecification (4) is considered for one-dimensional case in an example of certain chemical absorbing light at various frequencies in [3]. Also in the case of $d = 1$ the integral overspecification can be written in the following form:

$$\int_0^1 k(x)u(x,t)dx = E(t); \qquad o < t < T, \tag{5}$$

or

$$\int_0^{s(t)} u(x,t)dx = E(t); \qquad o < t < T, \tag{6}$$

where $k(x)$ and $s(t)$ are known functions. It is assumed that, for some constant $\rho > 0$, the kernel $k(x)$

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satisfies:

$$\int_0^1 |k(x)|dx \le \rho. \tag{7}$$

This problem appears in the mathematical modeling of many phenomena [1, 2]. Certain types of physical problems can be modeled by (1)-(4). For example if u is a temperature, then (1)-(4) can be regarded as a control problem finding the control $p(t)$. We want to identify the control function $p(t)$ that will yield a desired energy prescribed in a portion of the spatial domain.

The existence and uniqueness of the solution of this inverse problem is established under certain assumptions in [2, 4]. Also the theoretical discussion about this problem is found in [5]. In [11-13] the solution of this problem and similar higher dimensional problems are investigated. Some numerical methods are presented in [3, 13, 15] for solving this problem.

This inverse problem has many important applications. The interested readers can see [6-10]. In this paper, we propose two powerful methods to solve the discussed problem. The first is the HPM developed by He in [18, 19] and used in [20-24] among many others. The second is ADM developed by Adomian in [25, 26] and used heavily in the literature in [27-30] and the references therein. It is shown; Adomian decomposition method is equivalent to the homotopy perturbation method in the model problem. The two methods give rapidly convergent series with specific significant features for each scheme. The two methods, which accurately compute the solutions in a series form or in an exact form, are of great interest to applied sciences. The main advantage of the two methods is that it can be applied directly for all types of differential and integral equations, homogeneous or inhomogeneous. Another important advantage is that the methods are capable of greatly reducing the size of computational work while still maintaining high accuracy of the numerical solution. The effectiveness and the usefulness of both methods are demonstrated by finding exact solutions to the models that will be investigated. However, each method has its own characteristic and significance that will be examined.

This paper is arranged in the following manner. In Section 2, we present homotopy perturbation and Adomian decomposition methods. In Section 3, we apply the HPM and the ADM on the inverse parabolic problem with a control parameter. to present a clear overview of method, in Section 4, we implement these methods for finding the exact solution of a control parameter in one-, two- and three-dimensional parabolic equations. A conclusion is drawn in Section 5.

2. The methods

In what follows we will highlight briefly the main points of each of the two methods, where details can be found in [18-30].

2.1. Homotopy perturbation method

To illustrate the basic idea of this method [18-24], we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0; \qquad r \in \Omega. \tag{8}$$

Considering the boundary condition of:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0; \qquad r \in \Gamma, \tag{9}$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain $\Omega \subset \mathbb{R}^d; d = 1,2,3$. Generally speaking, the operator A can be divided into two parts which are L and N, where L is a simple part which is easy to handle and N contains the remaining parts of A.

Therefore equation (8) can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0. \tag{10}$$