ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 10, No. 2, 2015, *pp*. 169-172

Mathematical techniques to transform intuitionistic fuzzy multisets to fuzzy sets

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(*Received December 24, 2014, accepted February 27, 2015)*

Abstract. Intuitionistic fuzzy multiset introduced by Shinoj and Sunil is a relatively new research area. In this article, we proposed some mathematical techniques to convert intuitionistic fuzzy multisets to fuzzy sets for industrial uses such as in fuzzy logic and fuzzy control.

Keywords: fuzzy control, fuzzy logic, fuzzy sets, intuitionistic fuzzy sets, fuzzy multisets, intuitionistic fuzzy multisets, multisets.

1. Introduction

Zadeh [11] generalised fuzzy sets from classical sets theory by allowing intermediate situations between the whole and nothing. For a fuzzy set, a membership function is defined to describe the degree of membership of an element to a class. The membership value ranges from 0 to 1, where 0 shows that the element does not belong to a class, 1 means belongs, and other values indicate the degree of membership to a class. In fuzzy sets, the membership function replaced the characteristic function in crisp sets. A fuzzy set represents the elements in a class with the degree of membership to that class. Fuzzy set which is the extension of crisp set provides a means of representing and handling the vagueness of an object and imperfectly described knowledge.

However, the concept of fuzzy sets theory seems to be inconclusive because of the exclusion of non-membership function and the disregard for the possibility of hesitation margin. Atanassov critically studied these shortcomings and proposed a concept called intuitionistic fuzzy sets (IFSs) in [2-5]. This class (i.e. IFSs) incorporates both membership and non-membership functions with hesitation margin (i.e. neither membership nor non-membership functions).

Shinoj and Sunil [7-9] introduced intuitionistic fuzzy multisets (IFMSs) from the combination of IFSs and fuzzy multisets proposed by Yager [10]. In [6], IFMSs are considered as the generalisation of IFSs or the extension of fuzzy multisets.

2. Concise note on intuitionistic fuzzy multisets

Definition 1[11]: Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A=\{\langle x,\mu_A(x)\rangle : x\in X\}$, where $\mu_A(x): X\longrightarrow[0,1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.

Definition 2[4]: Let X be nonempty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form; $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$, where the functions $\mu_A(x),\nu_A(x) : X\longrightarrow[0,1]$ define the degree of membership and degree of non-membership of the element $x\in X$ to the set A. For every $x\in X$, $0\le\mu_A(x)+\nu_A(x)\le 1$.

Furthermore, $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$ is the intuitionistic fuzzy set index or hesitation margin and is the degree of indeterminacy concerning the membership of x in A, then $0\le\mu_A(x)+\nu_A(x)+\pi_A(x)\le 1$. Whenever $\pi_A(x)=0$, IFS reduces automatically to fuzzy set.

Definition 3[10]: Let X be a nonempty set. A fuzzy multiset (FMS) A drawn from X is characterized by a function, 'count membership' of A denoted by CM_A *s.t.* $CM_A: X\longrightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval $[0,1]$. Then for any $x\in X$, the value $CM_A(x)$ is a crisp multiset drawn from $[0,1]$. For each $x\in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^n(x))$, where $\mu_A^1(x) \geq \mu_A^2(x) \geq \ldots \geq \mu_A^n(x)$. Also, a fuzzy multiset A in X is a set of ordered sequence given as $A = \{\langle x, \mu_1(x), \mu_2(x), \mu_3(x), \ldots, \mu_n(x), \ldots \rangle : x \in X\}$, where $\mu_n(x): X \to [0,1]$ is the membership function of A.

Definition 4[7]: Let X be a nonempty set. An IFMS A drawn from X is characterized by two functions: "count membership" of A denoted as CM_A and "count non-membership" of A denoted as CN_A given respectively by $CM_A: X \longrightarrow Q$ and $CN_A: X \longrightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval $[0,1]$ s.t. for each $x \in X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CM_A(x)$ and it is denoted as $(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^n(x))$, where $\mu_A^1(x) \geq \mu_A^2(x) \geq \ldots \geq \mu_A^n(x)$ whereas the corresponding non-membership sequence of elements in $CN_A(x)$ is denoted by $(\nu_A^1(x), \nu_A^2(x), \ldots, \nu_A^n(x))$ *s.t.* $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ for every $x \in X$ and $i = 1, \ldots, n$.

This means, an IFMS A is defined as; $A = \{\langle x, CM_A(x), CN_A(x) \rangle : x \in X\}$ or $A = \{\langle x, \mu_A^i(x), \nu_A^i(x) \rangle : x \in X\}$, *for* $i = 1, \ldots, n$.

For each IFMS A in X, $\pi_A^i(x) = 1 - \mu_A^i(x) - \nu_A^i(x)$ is the intuitionistic fuzzy multisets index or hesitation margin of *x* in A. The hesitation margin $\pi_A^i(x)$ *for each* $i = 1, \ldots, n$ is the degree of non-determinacy of $x \in X$, to the set A and $\pi_A^i(x) \in [0,1]$. Similarly, $\pi_A^i(x)$ as in IFS, is the function that expresses lack of knowledge of whether $x \in A$ or $x \notin A$. Then, $\mu_A^i(x) + \nu_A^i(x) + \pi_A^i(x) = 1$ for each $i = 1, \ldots, n$.

Definition 5: We define IFMS alternatively. Let X be nonempty set. An IFMS A drawn from X is given as $A = \{\langle \mu_A^1(x), \ldots, \mu_A^n(x), \ldots, \boldsymbol{\nu}_A^1(x), \ldots, \boldsymbol{\nu}_A^n(x), \ldots \rangle x \in X\}$ where the functions $\boldsymbol{\mu}_A^i(x), \boldsymbol{\nu}_A^i(x): X \to [0,1]$ define the belongingness degrees and the non-belongingness degrees of A in X *s.t.* $0 \leq \boldsymbol{\mu}_A^i(x) + \boldsymbol{\nu}_A^i(x) \leq 1$ for $i = 1, \ldots$ If the sequence of the membership functions and non-membership (belongingness functions and non-belongingness functions) have only n-terms (i.e. finite), n is called the dimension of A. Consequently $A = \{\langle \mu_A^1(x), \ldots, \mu_A^n(x), \boldsymbol{\nu}_A^1(x), \ldots, \boldsymbol{\nu}_A^n(x) \rangle x \in X\}$ for $i = 1, \ldots, n$. when no ambiguity arises, we define $A = \{\langle \mu_A^i(x), \boldsymbol{\nu}_A^i(x) \rangle x \in X\}$ for $i = 1, \ldots, n$.

3. Our motivations

The need for the conversion of IFMSs to fuzzy sets arises when we have intuitionistic fuzzy multi-values (or data) and the available machine or Computer package can only accept fuzzy values. This is due to the fact that IFMS is relatively new and fuzzy set is industrially accepted for its efficiency and effective applications in computer programming and artificial intelligent (i.e. fuzzy logic) and fuzzy control.

The techniques

In converting IFMSs to fuzzy sets, one could be tempted to just neglect the hesitation margin (not even minding its proportions), but doing this will definitely present a deceptive result. We present some mathematical techniques which are effective and provide a pretty picture of IFMS in terms of fuzzy set. These techniques are adapted from Ansari *et al.* [1] work on the conversion of IFSs to fuzzy sets.

Let $X = \{x\}$ be nonempty set and let an IFMS A be given as $A = \{\langle x, (0.6, 0.3), (0.5, 0.3) \rangle : x \in X\}$. We are given $\mu_A^1(x) = 0.6$, $\mu_A^2(x) = 0.5$, $\nu_A^1(x) = \nu_A^2(x) = 0.3$; then $\pi_A^1(x) = 0.1$, $\pi_A^2(x) = 0.2$

Firstly we convert the IFMS to IFS by taking the mean values of each of the parameters. Then A becomes IFS as given thus;

$\mu_A(x) = 0.55$, $\nu_A(x) = 0.3$, and $\pi_A(x) = 0.15$