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The Method of Particular Solutions (MPS) for Solving One-Dimensional Hyperbolic Telegraph Equation

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Abstract. In this paper, the method of particular solution (MPS) is employed for the numerical solution of the one-dimensional (1D) telegraph equation based on radical basis functions (RBFs). Coupled with the time discretization and MPS, the proposed method is a truly meshless method which requires neither domain or boundary discretization. The algorithm is very simple so it is very easy to implement. The results of numerical experiments are presented, and are compared with analytical solutions to confirm the good accuracy of the presented scheme, the obtained numerical results also have been compared with the results obtained by some existing methods to verify the accurate nature of our method.

Keywords: method of particular solution (MPS), radical basis function (RBF), numerical solution, hyperbolic telegraph equation.

1. Introduction

This paper is devoted to the numerical computation of the one-dimension (1D) hyperbolic telegraph equation:

$$\frac{\partial^2 u}{\partial^2 t}+2\alpha\frac{\partial u}{\partial t}+\beta^2 u=\frac{\partial^2 u}{\partial^2 x}+f(x,t)\,,\ a\le x\le b\,,\ t>0\,, \tag{1.1}$$

with the initial conditions:

$$u(x,0)=h_0(x)\,,\ u_t(x,0)=h_1(x)\,,\ a\le x\le b\,, \tag{1.2}$$

and Dirichlet boundary conditions:

$$u(a,t)=g_0(t)\,,\ u(b,t)=g_1(t)\,,\ t>0 \tag{1.3}$$

where α and β are known constant coefficients, h_i and g_i ($i=0,1$) are known continuous functions. Both the electric voltage and the current in a double conductor, satisfy the telegraph equation, where x is distance and t is time. Note that, for $\alpha>0$, $\beta=0$, Eq. (1.1) represents a damped wave equation, and for $\alpha>\beta>0$, it is called telegraph equation [1].

The second-order telegraph equation with constant coefficients is commonly used in signal analysis for transmission and propagation of electrical signals [2] and also models mixture between diffusion and wave propagation by introducing a term that accounts for effects of finite velocity to standard heat or mass transport equation [3]. In fact the telegraph equation is more suitable than ordinary diffusion equation in modeling reaction diffusion for such branches of sciences. Moreover, this equation also has applications in other fields (see [4] and the references therein).

Recently, much attention has been given to the development, analysis, and implementation of stable methods for the numerical solution of second-order hyperbolic equations (see [5] and the reference therein). Mohanty et al [6,7], developed new three-level implicit unconditionally stable alternating direction implicit schemes for the two and three-space dimensional linear hyperbolic equations. These schemes are second-order accurate both in space and time. Dehghan and Shokri [8] solved the one-dimensional telegraph equation

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using Kansa's method. Z. W. Jiang, et al [9] extend this problem considered in [8] to one kind of partial differential equations with variable coefficients. A numerical method based on the interpolating scaling functions were described by Lakestani and N. Saray [10]. Evans and Hasan [11] applied an Alternating Group Explicit (AGE) method to obtain numerical solution of the telegraph equation. Marzieh Dosti, Alireza Nazemi [12,13] and J. Rashidinia1, et al [14] developed a numerical method using quartic B-spline collocation and cubic B-spline quasi-interpolation.

In this article, we present a new numerical scheme to solve the second-order hyperbolic telegraph equation using the Method of Particular Solutions (MPS) with the Thin Plate Splines (TPS) Radial Basis Function (RBF). The results of numerical experiments are presented, and are compared with analytical solutions to confirm the good accuracy of the presented scheme, the obtained numerical results also have been compared with the results obtained by some existing methods to verify the accurate nature of our method.

In last 25 years, the radial basis functions (RBFs) method is known as a powerful tool for scattered data interpolation problem. The use of RBFs as a meshless procedure for numerical solution of partial differential equations is based on the collocation scheme. Because of the collection technique, this method does not need to evaluate any integral. The main advantage of numerical procedures which use RBFs over traditional techniques is meshless property of these methods. RBFs are used actively for solving partial differential equations. The examples see [15,16]. In the last decade, the development of the RBFs as a truly meshless method for approximating the solutions of PDEs has drawn the attention of many researchers in science and engineering [17–19]. Meshless method has became an important numerical computation method, and there are many academic monographs are published [20–22].

The layout of the article is as follows : In section 2, we introduce the MPS method and apply this method on the hyperbolic telegraph equation. The results of numerical experiments are presented in section 3. Section 4 is dedicated to a brief conclusion. Finally, some references are introduced at the end.

2. The Method of Particular Solutions (MPS)

2.1. Radial basis function approximation

The approximation of a distribution $u(\boldsymbol{x})$, using RBF, may be written as a linear combination of N radial basis functions, usually it takes the following form:

$$u(\boldsymbol{x}) \approx \sum_{j=1}^{N} \lambda_j \varphi(\boldsymbol{x}, \boldsymbol{x}_j) + \psi(\boldsymbol{x}), \text{ for } \boldsymbol{x} \in \Omega \subseteq R^d \tag{2.1.1}$$

where N is the number of data points, $\boldsymbol{x} = (x_1, x_2, \cdots, x_d)$, d is the dimension of the problem, the λ 's are coefficients to be determined and φ is the radial basis function. Eq. (2.1.1) can be written without the polynomial ψ. In that case, φ must be unconditional positive definite to guarantee the solvability of the resulting system (e. g. Gaussian or Inverse Multiquadrics). However, ψ is usually required when φ is conditionally positive definite, i. e, when φ has a polynomial growth towards infinity. We will use the Thin Plate Splines (TPS), which defined as:

$$\text{TPS:} \quad \varphi(\boldsymbol{x}, \boldsymbol{x}_j) = \varphi(r_j) = r_j^{2m} \log(r_j), \quad m = 1,2,3,\cdots \tag{2.1.2}$$

where $r_j = \|\boldsymbol{x} - \boldsymbol{x}_j\|$ is the Euclidean norm.

If $\boldsymbol{P}_q^d$ denotes the space of d -variate polynomial of order not exceeding than q, and letting the polynomials $(P_1, P_2, \cdots, P_m)$ be the basis of $\boldsymbol{P}_q^d$ in R^d, then the polynomial $\psi(\boldsymbol{x})$ in Eq. (2.1.1) is usually written in the following form:

$$\psi(\boldsymbol{x}) = \sum_{i=1}^{m} \xi_i P_i(\boldsymbol{x}_j) \tag{2.1.3}$$

where $m = (q-1+d)!/(d!(q-1)!)$. To get the coefficients $(\lambda_1, \lambda_2, \cdots, \lambda_N)$ and $(\xi_1, \xi_2, \cdots, \xi_m)$, the collocation method is used. However, in addition to the N equations resulting from collecting Eq. (2.1.1) at