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Finite-time synchronization of time-delay Hindmarsh- Rose system with external disturbance

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Abstract. In this paper, the finite-time synchronization of time-delay Hindmarsh-Rose with external disturbance is investigated. Based on Lyapunov stability theory, a scheme is proposed and controllers are constructed to realize it. Finally， numerical simulations are given to verify the theoretical results.

Keywords: Finite-time synchronization, Hindmarsh-Rose system, time-delay, external disturbance

1. Introduction

Recently, chaos synchronization has received much attention [1-5]. Many kinds of methods have been proposed to study the synchronization of chaotic system, such as drive-response synchronization method [6], adaptive control method [7], backstepping method [8], and so on. Various synchronization have been observed, such as complete synchronization [9], Lag synchronization [10], phase synchronization [11], etc. With further investigation, the time to realize synchronization received attention. Therefore finite-time synchronization is studied in many fields [12-17], especially in neuron system.

Experimental studies [18] suggested that synchronization has significant meaning in the information transferring of neurons. Meanwhile, in information transformation among neurons, not only the time-delay always exists, but also the external disturbance is inevitable. Therefore, it is necessary to investigate the synchronization of time-delay neural system with disturbance.

In this paper, finite-time synchronization of time-delay Hindmarsh-Rose system with external disturbance is to be explored. Other parts are arranged as follows. Section 2 gives some preliminaries. In section 3, a scheme is descripted to realize finite-time synchronization of time- delay Hindmarsh-Rose system with disturbance. Section 4 gives some numerical simulations. Result is given in Section 5.

2. Preliminaries

$$\dot{x} = f(x) + F(x)\alpha + d(x,t), \tag{1}$$
$$\dot{y} = g(y) + G(y)\beta + u(x,t), \tag{2}$$

where x, $y \in R^n$ are state vectors. $f(x), g(y) \in R^{n+1}$ are linear matrix functions. $F(x), G(y) \in R^{n+1}$ are nonlinear matrix functions. α, β are parameter vectors. $d(x,t) \in R^n$ is external disturbance. $u(x,t)$ is controller vector.

Hypothesis 1(H1): The nonlinear matrix function $g(x)$ satisfies Lipschitz condition, that is

$$\|g(x) - g(y)\| \le L_g \|x - y\|,$$

where L_g is an appropriate positive constant. $\|\cdot\|$ denotes the norm of matrix or vector, defined as $\|A\| = (\sum_{j=1}^{m}\sum_{i=1}^{n} {a_{ij}}^2)^{\frac{1}{2}}$ for matrix $A = (a_{ij})_{m\times n}$ or $\|x\| = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}}$ for vector $x = (x_1, \cdots, x_n)$.

Hypothesis 2(H2): The uncertain parameters α, β and disturbance $d(x,t)$ are all bounded in terms of norm, namely, there exist positive constants $\theta_\alpha, \theta_\beta, \theta_d$ such that

Let $e = y - x$, subtracting (1) from (2) yields

$$\dot{e} = \dot{y} - \dot{x} = g(y) - f(x) + G(y)\beta - F(x)\alpha + u(x,t) - d(x,t). \tag{3}$$

Therefore, to realize the finite-time synchronization of systems (1) and (2) means to obtain finite-time stability of error system (3). For this end, following definition of finite-time synchronization and some necessary lemmas are introduced as follows.

Definition 1[19] Consider two chaotic systems

$$\dot{x}_m = f(x_m),$$

$$\dot{x}_s = h(x_s), \tag{4}$$

where x_m, x_s are two n-dimensional vectors. The subscripts 'm' and 'n' stand for the master and slave systems, respectively. $f: R^n \to R^n$ and $h: R^n \to R^n$ are vector-valued functions. If there is a positive constant T such that

$$\lim_{t\to T}\|x_m - x_s\| = 0,$$

and $\|x_m - x_s\| \equiv 0$ if $t \geq T$, then it is said that the finite-time synchronization between two systems of (4) can be achieved.

Lemma 1[20] Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -cV^{\eta}(t),\ \forall t \geq t_0, V(t_0) \geq 0, \tag{5}$$

where $c > 0$, $0 < \eta < 1$ are all constants. Then for any given t_0, $V(t)$satisfies following inequality:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ \ t_0 \leq t \leq t_1, \tag{6}$$

and

$$V(t) \equiv 0, \forall t \geq t_1 \tag{7}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{8}$$

Proof. Consider differential equation:

$$\dot{X}(t) = -cX^{\eta}(t), X(t_0) = V(t_0). \tag{9}$$

Although equation (9) doesn't satisfy the global Lipschitz condition, the unique solution of it can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t-t_0). \tag{10}$$

Therefore, from the comparison Lemma [20], it can be gotten that

$$V^{1-\eta} \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ \ t_0 \leq t \leq t_1, \tag{11}$$

and

$$V(t) \equiv 0, \forall t \geq t_1 \tag{12}$$

with t_1 given in (8).

Lemma 2[21] Suppose $0 < r \leq 1$, a, b are positive constants, then the following inequality is quite straightforward:

$$(|a| + |b|)^r \leq |a|^r + |b|^r.$$

3. Finite-time synchronization of time-delay Hindmarsh-Rose system with disturbance

In this paper, Hindmarsh-Rose (HR) system with time-delay is considered as following:

$$\begin{aligned} \dot{x} &= ax^2 - bx^3 + y - z(t-\tau) + I_{ext}, \\ \dot{y} &= c - dx^2 - y, \\ \dot{z} &= r(S(x+k) - z), \end{aligned} \tag{13}$$

where $\tau > 0$ is the time delay. When$\tau = 0$, model (13) is a mathematical representation of the firing behavior of neuron proposed by Hindmarsh and Rose [22]. In system (13), the variables x, y and z represent the membrane potential of the neuron, the recovery variable, and the adaptation current, respectively. The current I_{ext} represents an external influence on the system. a, b, c, d, r, S, k are real constants.

Model (13) may describe regular bursting or chaotic bursting for certain domains of the parameters. When $\tau = 1$, other parameters are chosen as $a = 3.0$, $b = 1.0$, $c = 1.0$, $d = 5.0$, $r = 0.006$, $S = 4.0$, $k = 1.6$, system (13) can show various complex dynamical behaviors with the changing of I_{ext}. For example, when $I_{ext} = 2.6$ and $I_{ext} = 3.1$, system (13) is regular bursting (Fig.1) and chaotic bursting (Fig.2), respectively.