

Finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system

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Abstract. In this paper, based on Lyapunov stability theory, a scheme to realize finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system is proposed. By theoretical analysis, the controllers are constructed. Finally, numerical simulations are given to verify the theoretical results.

Keywords: Finite-time hybrid synchronization, hyperchaotic Lorenz system, time-delay

1. Introduction

In the past few years, synchronization of chaotic system drew much attention of researchers because of its potential applications in many fields. Many kinds of synchronization have been investigated [1-5]. Accordingly, many effective schemes have been proposed [6-11]. With further research on synchronization, more and more people have realized the importance of the time in achieving synchronization. For this end, some methods have been proposed to investigate finite-time synchronization because of its showing the robustness and disturbance rejection properties of system [12]. Therefore, finite-time synchronization has been widely studied [13-18].

Based on the existing results, finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system is investigated in this paper. Other parts of this paper are arranged as follows. Section 2 gives some preliminaries. In Section 3, the scheme to realize the finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system is proposed. Simulation results are given in Section 4. Conclusion is drawn in Section 5.

2. Preliminaries

Definition 1. Suppose that x, y are n -dimensional state vectors and $f: R^n \rightarrow R^n, g: R^n \rightarrow R^n$ are vector-valued functions. Considering two chaotic systems as follows:

$$\begin{aligned}\dot{x} &= f(x), \\ \dot{y} &= g(y).\end{aligned}\tag{1}$$

If there exists a positive constant T such that

$$\lim_{t \rightarrow T} \|x - y\| = 0,$$

and $\|x - y\| \equiv 0$ when $t \geq T$, then it is said that the two systems of (1) can achieve finite-time synchronization.

Lemma 1[19]. Assume that a continuous, positive-definite function $V(t)$ satisfies following differential inequality:

$$\dot{V}(t) \leq -cV^\eta(t), \forall t \geq t_0, V(t_0) \geq 0,\tag{2}$$

where $c > 0, 0 < \eta < 1$ are all constants. Then for any given $t_0, V(t)$ satisfies following inequality:

$$V^{1-\eta} \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0), t_0 \leq t \leq t_1,\tag{3}$$

and

$$V(t) \equiv 0, \forall t \geq t_1,\tag{4}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}.\tag{5}$$

Proof. Consider differential equation:

$$\dot{X}(t) = -cX^\eta(t), X(t_0) = V(t_0).\tag{6}$$

Although equation (6) doesn't satisfy the global Lipschitz condition, the unique solution of it can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t-t_0).\tag{7}$$

Therefore, from the comparison Lemma [20], it can be gotten that

$$V^{1-\eta} \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0), t_0 \leq t \leq t_1,$$

and

$$V(t) \equiv 0, \forall t \geq t_1$$

with t_1 given in (5).

Lemma 2[21]. Suppose $0 < r \leq 1$, a, b are all positive numbers, then the following inequality is quite straightforward:
 $(|a| + |b|)^r \leq |a|^r + |b|^r$. (8)

3. Finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system

In this section, hyperchaotic Lorenz system with time-delay is considered as following:

$$\dot{x} = a(y - x) + rw(t - \tau),$$

$$\dot{y} = cx - y - xz,$$

$$\dot{z} = xy - bz,$$

$$\dot{w} = -yz - dw, \quad (9)$$

where $\tau > 0$ is time delay. When $\tau = 0$, system (9) is hyperchaotic Lorenz system [22]. If $a = 10, b = 8/3, c = 28, d = 1, \tau = 1$, system (9) has hyperchaotic behavior with two positive Lyapunov exponents[23] $\lambda_1 = 0.6513, \lambda_2 = 0.1394$. The hyperchaotic attractors are shown in Fig. 1 (3D overview).

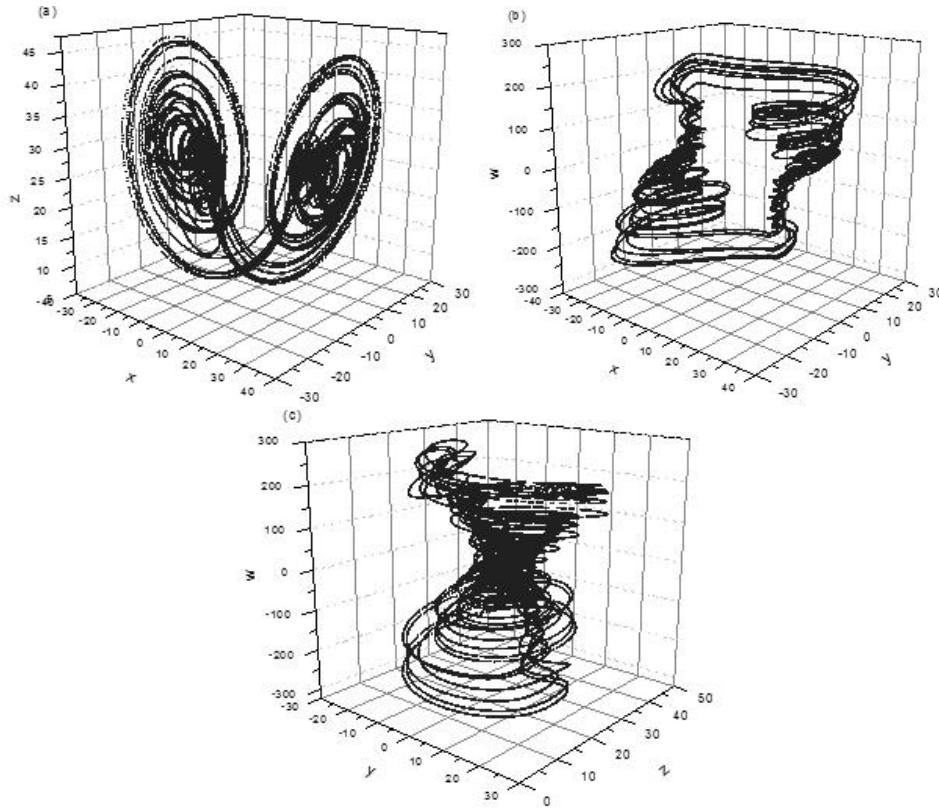


Fig. 1. Hyperchaotic attractors of system (9) (a) (x, y, z) , (b) (x, y, w) , (c) (y, z, w) .

The drive system is given as (10):

$$\dot{x}_1 = a(y_1 - x_1) + rw_1(t - \tau),$$

$$\dot{y}_1 = cx_1 - y_1 - x_1z_1,$$

$$\dot{z}_1 = x_1y_1 - bz_1,$$

$$\dot{w}_1 = -y_1z_1 - dw_1, \quad (10)$$

and the response system is written as (11):

$$\dot{x}_2 = a(y_2 - x_2) + rw_2(t - \tau) + u_1,$$