

MORE-FOR-LESS PARADOX IN A SOLID TRANSPORTATION PROBLEM

AtanuDas^a, Debi Prasad Acharya^b and ManjusriBasu^a

^aDepartment of Mathematics; University of Kalyani; Kalyani - 741235; W. B.; India.

^bDepartment of Mathematics; NabadwipVidyasagar College; Nabadwip; Nadia; W.B.; India.

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Abstract. In this paper, we discuss more-for-less paradox in a solid transportation problem. Thereby, we demonstrate a theorem which gives a comfortable condition for the existence of paradox in this type of problem. Next we present an algorithm to find out all the paradoxical pairs as well as paradoxical range of flow and paradoxical pair for a specified flow if paradox exists. Also we illustrate a numerical example in support of the given algorithm.

Keywords: Solid Transportation Problem, Paradox in a Solid Transportation Problem, Paradoxical Range of Flow

1. Introduction

The solid transportation problem(STP) is a generalization of the classical transportationproblem(TP). Haley [13] was the pioneer in this field. The STP includes three types of constraints viz., source constraint, destination constraint and capacity (e.g., conveyance, various types of products, etc.) constraint instead of two constraints viz., source constraint and destination constraint in classical TP. The STP has an immense application in real life problems. Matvenco[14]considered combinatorial approach to the problem of solvability of the solid (three-index) transportation problem. Gen et al. [11] considered the STP in fuzzy environment. Basuet al.[8] gave an algorithm for obtaining the optimum time-cost trade-off in STP. They[9]also developed an algorithm for finding the optimum solution of solid fixed charge transportation problem. Basu and Acharya[5] developed an algorithm for the optimum time-cost trade-off in generalized STP. They [6] also considered on quadratic fractional generalized bi-criterion STP. Yang and Feng[20] studied on bi-criteria STP with fixed charge under stochastic environment. Ojhaet al.[15] considered bi-criterion an entropy based STP under fuzzy environment. They [16] also developed STP for an item with fixed charge, vechicle cost and price discounted varying charge using genetic algorithm. Tao and Xu[18] considered a class of rough multiple objective programming and its application to STP.

In some cases of the classical TP, an increase in the supplies and demands or in other words, increase in the flow results a decrease in the optimum transportation cost. This type of behavior which means paradoxical, is called transportation paradox. Basically, the papers of Charnes and Klingman[10] and Szwarc[17] are treated as the sources of transportation paradox for the researchers. In the paper of Charnes and Klingman, they name it "more-for-less" paradox and wrote "The paradox was first observed in the early days of linear programming history (by whom no one knows) and has been a part of the folklore known to some (e.g. A.Charnes and W.W.Cooper), but unknown to the great majority of workers in the field of linear programming". Subsequently, in the paper of Appa[4], he mentioned that this paradox is known as "Doig Paradox" at the London School of Economics, named after Alison Doig. Gupta et al.[12] established a sufficient condition for a paradox in a linear fractional transportation problem with mixed constraints. Adlakha and Kowalski [3] derived a sufficient condition to identify the cases where the paradoxical situation exists. Deinekoet al.[19] developed a necessary and sufficient condition for a cost matrix which is immuned against the transportation paradox. Basuet. al.[7] provided an algorithm for obtaining paradoxical range of flow and paradoxical flow for a specified flow.Acharyaet al. [1] developed an algorithm for obtaining paradoxical range of flow and paradoxical flow for a specified flow in a fixed charge transportation problem. They [2] also considered paradox in a fuzzy transportation problem with linear constraints.

In this paper, we present a method for solving solid transportation problem with linear constraints. Thereby, we state a sufficient condition for existence of paradox. Then we give an algorithm for obtaining all

paradoxical pairs, paradoxical range of flow and paradoxical pair for a specified flow in such type of problem. We also justify the theory by illustrating a numerical example.

2. Problem Formulation

In this paper, our goal is to obtain a transportation plan which satisfies all required demands and minimizes the overall transportation cost of the problem:

$$P_1 : \text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l p_{ijk} x_{ijk}$$

subject to the constraints,

$$\begin{aligned} \sum_{j=1}^n \sum_{k=1}^l x_{ijk} &= a_i \quad \forall i \in I = (1, 2, 3, \dots, m) \\ \sum_{i=1}^m \sum_{k=1}^l x_{ijk} &= b_j \quad \forall j \in J = (1, 2, 3, \dots, n) \\ \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &= c_k \quad \forall k \in K = (1, 2, 3, \dots, l) \end{aligned}$$

$$\text{and } x_{ijk} \geq 0 \quad \forall (i, j, k) \in I \times J \times K$$

where,

x_{ijk} = the amount of k -th type of product transported from the i -th origin to the j -th destination,

p_{ijk} = the cost involved in transporting per unit of the k -th type of product from the i -th origin to the j -th destination,

a_i = the number of units available at the i -th origin,

b_j = the number of units required at the j -th destination,

c_k = requirement of the number of units of the k -th product.

Hence this problem consists of m origins and n destinations along with l types of products. We assume that $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = \sum_{k=1}^l c_k$ and $a_i, b_j, c_k > 0$ for all i, j, k which is known as balanced STP. If the STP P_1 is unbalanced then we convert it to a balanced STP by using dummy variables with zero cost.

Let $X^\alpha = \{x_{ijk}^\alpha | (i, j, k) \in I \times J \times K\}$ be a basic feasible solution corresponding to the basis B of the problem P_1 , the corresponding value of the objective function $Z^\alpha = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l p_{ijk} x_{ijk}^\alpha$ and the flow $F^\alpha = \sum_{i \in I} a_i = \sum_{j \in J} b_j = \sum_{k \in K} c_k$

Definition 2.1. The pair (Z^α, F^α) is called the **cost-flow pair** corresponding to the feasible solution X^α .

Condition of optimality: The condition of optimality of the problem P_1 is $p'_{ijk} = (u_i + v_j + w_k) - p_{ijk} \leq 0$ for all $(i, j, k) \notin B$, where u_i, v_j, w_k are the dual variables corresponding to the basis B such that $p_{ijk} = (u_i + v_j + w_k)[11]$.

Let $Z^0 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l p_{ijk} x_{ijk}^0$ and $F^0 = \sum_{i \in I} a_i = \sum_{j \in J} b_j = \sum_{k \in K} c_k$ be the optimum cost and flow respectively corresponding to the optimum solution $X^0 = \{x_{ijk}^0 | (i, j, k) \in I \times J \times K\}$.

Definition 2.2 In a STP, if we can obtain flow $F^1 > F^0$ with cost $Z^1 < Z^0$ then we say **paradox** occurs.

Definition 2.3 The pair (Z^1, F^1) is called the **paradoxical cost-flow pair** if paradox exists.

Definition 2.4 A cost-flow pair (Z^2, F^2) is called an **improved paradoxical cost-flow pair** if $Z^2 < Z^1$ and $F^2 > F^1$.

Definition 2.5 The paradoxical cost-flow pair (Z^*, F^*) where (Z^i, F^i) , $1 < i < g$, such that $Z^i < Z^{i-1}$ and $F^i > F^{i-1} \forall i$ be all paradoxical cost-flow pairs and $Z^* = Z^g$ and $F^* = F^g$, is called the **best paradoxical cost-flow pair**.