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A New Rational Quadratic Trigonometric Bézier Curve with Three Shape Parameters

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Abstract. A rational quadratic trigonometric Bézier curve with three shape parameters is presented in this work. The properties of the curve are studied. The C^0, C^1 and C^2 continuous conditions for joining two constructed curves are discussed. The shape of the curve can be flexibly controlled with shape parameters and weight without changing the control points. Some examples are given.

Keywords: quadratic trigonometric basis functions, rational quadratic trigonometric Bézier curve, shape parameter, continuity.

1. Introduction

Trigonometric splines have gained widespread application in various fields of mathematics, physics and engineering, in particular in curve design (cf. [1-6]). In recent years, Bézier form of trigonometric curvers with shape parameters has received very much attention in Computer Aided Geometric Design (CAGD). For example, Han [7-8] proposed quadratic trigonometric Bézier curves and cubic trigonometric Bézier curves with a shape parameter. Han et al [9] presented the cubic trigonometric Bézier curve with two shape parameters. Sheng et al [10] introduced the quasi-quartic Bézier-type curves with parameter α . Bashir et al [11] gave a class of quasi-quintic trigonometric Bézier curve with two shape parameters.

In this paper, we define a new rational quadratic trigonometric Bézier curve with three shape parameters. It is more flexible to control the shape than the presented curve in [12]. The composition of two curve segments using C^0, C^1 and C^2 continuity conditions is discussed. Some examples illustrate that the constructed curve in this paper provides an effective method for designing curves and geometric modeling.

2. Basis Functions

Definition 1. For $t \in [0,1]$, the quadratic trigonometric basis functions with three shape parameters α, β and γ are defined as

$$\begin{cases} b_0(t) = (1-\sin\frac{\pi}{2}t)(1-\alpha\sin\frac{\pi}{2}t)(1-\beta\sin\frac{\pi}{2}t) \\ b_1(t) = 1 - b_0(t) - b_2(t) \\ b_2(t) = (1-\cos\frac{\pi}{2}t)(1-\alpha\cos\frac{\pi}{2}t)(1-\gamma\cos\frac{\pi}{2}t) \end{cases} \tag{1}$$

where $\alpha, \beta, \gamma \in [-1,1]$ and satisfy that α with β and γ cannot be simultaneously negative.

Theorem 1. The basis functions have the following properties:

(i) Non-negativity: $b_i(t) \geq 0$, $i = 0,1,2$.

(ii) Partitin of unity: $\sum_{i=0}^{2} b_i(t) \equiv 1$.

(iii) Symmetry: $b_i(t;\alpha,\beta,\gamma) = b_i(1-t;\alpha,\gamma,\beta)$, $i = 0,1,2$.

(iv) Monotonicity: For fixed $t \in [0,1]$, $b_0(t)$ is monotonically decreasing for shape parameters α and β. $b_2(t)$ is monotonically decreasing for shape parameters α and γ. $b_1(t)$ is monotonically increasing respectively for shape parameters α, β and γ.

(v) Properties at the endpoints:

$$b_0(0)=1,\quad b_1(0)=0,\quad b_2(0)=0;$$

$$b_0(1)=0,\quad b_1(1)=0,\quad b_2(1)=1;$$

$$b_0^{'}(0)=-\frac{\pi}{2}(1+\alpha+\beta),\quad b_1^{'}(0)=\frac{\pi}{2}(1+\alpha+\beta),\quad b_2^{'}(0)=0;$$

$$b_0^{'}(1)=0,\quad b_1^{'}(1)=-\frac{\pi}{2}(1+\alpha+\gamma),\quad b_2^{'}(0)=\frac{\pi}{2}(1+\alpha+\gamma).$$

Proof. The results immediately follow from the defintion of the basis functions (1).

Fig. 1 shows the curves of the quadratic trigonometric basis functions for $\alpha=0,\beta=-1,\gamma=1$ (solid lines), for $\alpha=1,\beta=0,\gamma=0$ (short dashed lines), and $\alpha=-1,\beta=1/2,\gamma=1$ (long dashed lines), respectively.

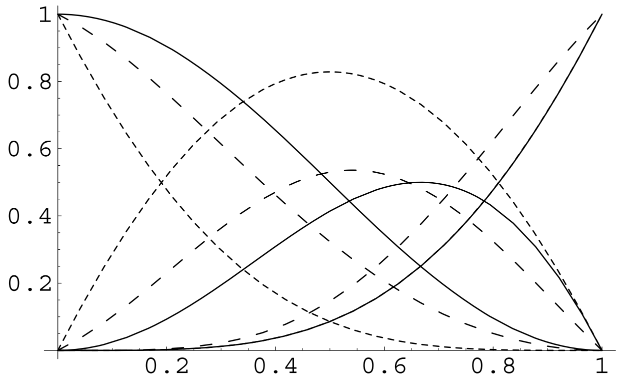

Fig. 1: The quadratic trigonometric basis functions.

3. The RQTB Curve

Definition 2. Given that $P_i (i=0,1,2)$ are three control points in $R^d (d=2,3)$,

$$R(t)=\frac{b_0(t)P_0+b_1(t)P_1\omega+b_2(t)P_2}{b_0(t)+b_1(t)\omega+b_2(t)},\quad t\in[0,1] \tag{2}$$

is called the rational quadratic trigonometric Bézier (RQTB, for short) curve with three shape parameters α, β and γ, where $\omega(>0)$ is called the weight of the function, and the basis functions $b_i(t)$ are defined as (1).

Theorem 2. The RQTB curve has the following properties:

(i) Terminal properties: