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Intuitionistic Fuzzy Linear and Quadratic Equations

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Abstract. In this paper we have considered simple linear and quadratic equations in Intuitionistic Fuzzy Environment. Here coefficients of the equations are taken as Generalized Triangular Intuitionistic Fuzzy Numbers. We have used the strong and weak solution concept to solve these Intuitionistic Fuzzy Equations. Solution procedures have shown in details.

Keywords: Inuitionistic Fuzzy Linear Equations, Inuitionistic Fuzzy Quadratic Equations, Generalized Triangular Intuitionistic Fuzzy Number (GTIFN), strong and weak solutions.

1. Introduction

Atanassov[1] introduced the concept of Intuitionistic Fuzzy Sets (IFS), which is a generalization of the concept of fuzzy set. In uncertain environments, the modeled equation can be an Intuitionistic Fuzzy Equation. The solution for Intuitionistic Fuzzy Equations plays an important role in uncertain decision making.

In literature, standard analytical techniques are proposed by Buckley and Qu [7,8,11]. Buckley [6,7] considered the solution of linear fuzzy equations using Classical methods, Zadeh's extension principle and the concept of fuzzy numbers and arithmetic operations on it introduced by Zadeh [14,15]. But we see that the solutions of Zadeh's extension principle method and interval arithmetic methods do not satisfy the given fuzzy equations always. In our previous paper [19] we have used another approach (strong and weak solution concept) to solve fuzzy linear and quadratic equations.

In this paper we have solved Intuitionistic Fuzzy Linear and Quadratic Equations by using the concept of strong and weak solution and coefficients are taken as a Generalized Triangular Intuitionistic Fuzzy Numbers. We have also solved two problems following Buckley,Qu[7] using this concept in this paper.

2. Preliminary concepts

Definition-2.1: Intuitionistic Fuzzy Sets: Let $U = \{x_1, x_2, \dots, x_n\}$ be a finite universal set. An Intuitionistic Fuzzy Set $\tilde{A}^i$ in a given universal set U is an object having the form

$$\tilde{A}^i = \left\{\langle x_i, \mu_{\tilde{A}^i}(x_i), \upsilon_{\tilde{A}^i}(x_i)\rangle : x_i \in U\right\}$$

Where the functions

$\mu_{\tilde{A}^i}: U \to [0,1]$; i.e. , $x_i \in U \to \mu_{\tilde{A}^i}(x_i) \in [0,1]$

and $\upsilon_{\tilde{A}^i}: U \to [0,1]$; i.e. , $x_i \in U \to \upsilon_{\tilde{A}^i}(x_i) \in [0,1]$

define the degree of membership and the degree of non-membership of an element $x_i \in U$, such that they satisfy the following conditions:

$$0 \le \mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i) \le 1, \forall x_i \in U$$

which is known as Intuitionistic Condition. The degree of acceptance $\mu_{\tilde{A}^i}(x_i)$and of non-acceptance $\upsilon_{\tilde{A}^i}(x_i)$ can be arbitrary.

Definition-2.2: $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-cuts: A set of (α, β)-cut, generated by IFS $\tilde{A}^i$, where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \le 1$ is defined as

$$\tilde{A}^i{}_{\alpha,\beta} = \begin{cases} \left(x, \mu_{\tilde{A}^i}(x), \upsilon_{\tilde{A}^i}(x)\right); & x \in U \\ \mu_{\tilde{A}^i}(x) \ge \alpha, \upsilon_{\tilde{A}^i}(x) \le \beta; & \alpha, \beta \in [0,1] \end{cases},$$

where (α, β)-cut, denoted by $\tilde{A}^i{}_{\alpha,\beta}$, is defined as the crisp set of elements x which belong to $\tilde{A}^i$ at least to the degree α and which does belong to $\tilde{A}^i$ at most to the degree β.

Definition-2.3: Intuitionistic Fuzzy Number: An Intuitionistic Fuuzy Number $\tilde{A}^i$ is

i. An Intuitionistic Fuuzy Subset on the real line
ii. Normal i.e. there exists $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}^i}(x_0) = 1$(so $\upsilon_{\tilde{A}^i}(x_0) = 0$)
iii. Convex for the membership function $\mu_{\tilde{A}^i}$ i.e.

$$\mu_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda x_2)) \geq \min\{\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\}; \forall\, x_1, x_2 \in \mathbb{R}\ , \lambda \in [0,1]$$

iv. Concave for the non-membership function $\upsilon_{\tilde{A}^i}$ i.e.

$$\upsilon_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda x_2)) \leq \max\{\upsilon_{\tilde{A}^i}(x_1), \upsilon_{\tilde{A}^i}(x_2)\}; \forall\, x_1, x_2 \in \mathbb{R}\ , \lambda \in [0,1]$$

Definition-2.4: Generalized Triangular Intuitionistic Fuzzy Number: A Generalized Triangular Intuitionistic Fuzzy Number (GTIFN) is denoted by $\tilde{A}^i = (a_1, a_2, a_3; w, u)$ is a special Intuitionistic Fuzzy Set on a real number set $\mathbb{R}$, whose membership function and non-membership functions are defined as

$$\mu_{\tilde{A}^i}(x) = \begin{cases} 0 & x \leq a_1 \\ w\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ w\ , & x = a_2 \\ w\frac{a_3-x}{a_3-a_2}\ , & a_2 \leq x \leq a_3 \\ 0\ , & a_3\ \leq x \end{cases}, \upsilon_{\tilde{A}^i}(x) = \begin{cases} 1 & x \leq a_1 \\ 1-(1-u)\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ u\ , & x = a_2 \\ 1-(1-u)\frac{a_3-x}{a_3-a_2}\ , & a_2 \leq x \leq a_3 \\ 1\ , & a_3\ \leq x \end{cases}$$

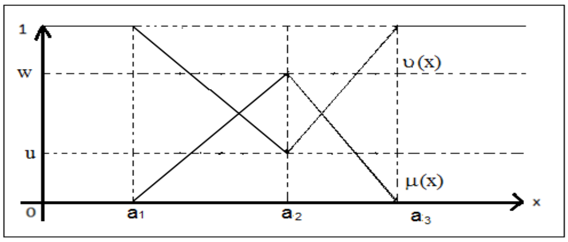

Fig-2.1: Rough sketch of membership and non-membership functions of the above GTFN

where w and u represent the maximum degree of membership and minimum degree of non-membership respectively such that they satisfy the condition

$$0 \leq w \leq 1, 0 \leq u \leq 1 \text{ and } 0 \leq w + u \leq 1$$

Note: GTIFN $(a_1, a_2, a_3; w, u) \xrightarrow{w=1-u}$ GTFN $(a_1, a_2, a_3; w) \xrightarrow{w=1,\ u=0}$ TFN (a_1, a_2, a_3)

Definition 2.5: Equality of two GTIFNs: Two GTIFNs $\tilde{A}^i = (a_1, a_2, a_3; w_1, u_1)$ and $\tilde{B}^i = (b_1, b_2, b_3; w_2, u_2)$ are equal when $a_1 = b_1, a_2 = b_2, , a_3 = b_3$ and $w_1 = w_2, u_1 = u_2$.

Definition2.6: Let $\tilde{A}^i$ =$(a_1, a_2, a_3; w_a, u_a)$ and $\tilde{B}^i$ =$(b_1, b_2, b_3; w_b, u_b)$ be two positive GTIFNs.
Let $w = min(w_a, w_b)$ and $u = max(u_a, u_b)$where $0 < w, u \leq 1$and $0 < w + u \leq 1$

(i) The addition of two GTIFNs $\tilde{A}^i, \tilde{B}^i$ is another GTIFN

$$\tilde{C}^i = (a_1 + b_1, a_2 + b_2, a_3 + b_3; w, u)$$

(ii) The subtraction of two GTIFNs $\tilde{A}^i, \tilde{B}^i$is another GTIFN

$$\tilde{C}^i = (a_1 - b_3, a_2 - b_2, a_3 - b_1; w, u)$$

(iii) The multiplication of two GTIFNs $\tilde{A}^i, \tilde{B}^i$ is a Generalized Triangular Shaped Intuitionistic Fuzzy Number (GTsIFN) $\tilde{C}^i \approx (a_1 b_1, a_2 b_2, a_3 b_3; w, u)$

(iv) The division of two GTIFNs $\tilde{A}^i, \tilde{B}^i$ is a GTsIFN $\tilde{C}^i \approx \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}; w, u\right)$

Definition 2.7: A Generalized Intuitionistic Fuzzy Number is completely determined by the pair $\langle [X_{\mu_L}(\alpha), X_{\mu_R}(\alpha)], [X_{\upsilon_L}(\beta), X_{\upsilon_R}(\beta)] \rangle$ of functions $X_{\mu_L}(\alpha), X_{\mu_R}(\alpha), X_{\upsilon_L}(\beta), X_{\upsilon_R}(\beta)$, $0 \leq \alpha \leq w, u \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1$ which satisfy the following requirements:

1. $X_{\mu_L}(\alpha)$ is a bounded monotonic increasing left continuous function over $[0, w]$.

$$\text{i.e.} \frac{d}{d\alpha}[X_{\mu_L}(\alpha)] > 0$$

2. $X_{\mu_R}(\alpha)$ is a bounded monotonic decreasing left continuous function over $[0, w]$.

$$\text{i.e.} \frac{d}{d\alpha}[X_{\mu_R}(\alpha)] < 0$$