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Local Stability and Hopf Bifurcation Analysis for a Predator-Prey control Model with Two Delays

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Abstract. Based on facts of time delay will influence dynamical behavior and control theory is more appropriate to reflect the natural rule, constructed a predator-prey control model with two delays,choose the two delays as the bifurcation parameter,according to the Routh-Hurwize discriminant method,we studied the local stability and Hopf branch of the control system.The sufficient conditions for the local stability and the existence of Hopf bifurcation are established.Finally,the effectiveness of the controller is verified by numerical simulation.

Keywords: Predator-prey model, stability analysis, Hopf bifurcation, time delay, control

1. Introduction

Time delay is often appear in biological activity,the fact that tie delay will influence dynamical behavior to reflect the nature rule.In the population ecology,since May find would destroy the stability of the logistic model's positive equilibrium[1],there has been a large number of literature [2-3],to study the effect of delay on the stability of positive equilibrium in ecological models.In biological systems,as the gestation process of species,species of digestion and transformation process as well as their mature time and so on, there are the time delay phenomenon.The existence of delays tends to evolve more complex dynamical properties in the system.If we can fully understand and master delay affects the dynamical property of ecological system,you can use the master of laws to do better a good job of prevention and control,protect the rare resources, maintaining ecological balance.

In the nonlinear biological time delay dynamical system,there is the most discussed is Hopf bifurcation.There has been some researchers already studied the Hopf bifurcation of ecosystem.Faria[4] consider hunting time delay, and regarding time delay as the bifurcation parameter study the stability of system and bifurcation.Yan hezhang[5] comprehensive two time delay,assuming two time delays are equal,the Hopf bifurcation of system is researched.

There have been research on time delay dynamical systems about Hopf bifurcation,but most of them are about 2d[6,7],about the higher time delay dimension systems's Hopf bifurcation and bifurcation control's research is relatively few.Yongli song et al [11,12,13] studied a few class with multi-delay prey system,the sufficient conditions for the local stability and the exitence of Hopf bifurcation are established.Tang C B,Chen Y Y. et al[14,15] respectively research three dimensional predator-prey systems with time delay, discussed the conditions of Hopf bifurcation near the positive equilibrium,and directive of the bifurcation is given.Fan meng et al in article [16]and[17] respectively discussed the Lotka-Volterra system with n kinds of competition and with feedback control system.Jiang Guirong and Lu Qishao [18]research the dynamics of predator-prey system behavior with state feedback control.

Considering the different biological mature time and digesting time are different,in this paper,considering the consider of two delays,in addition,on the study of ecological system's control, main consideration is state feedback control previously.For example, in order to eliminate blooms occurred,an effective way is to introduce feedback control variables in the equation (such as silver carp, bighead) to change the equilibrium of the system.In fact,intraspecific effect coefficient and interaction effect coefficient as parameters influenced by many factors such as temperature and sunshine etc,so also by changing the parameters to adjust system.In this paper, considering parameters and state feedback control,put forward the following predator-prey control model with two delays.

The remainder of this paper is organized as follows:the control model is proposed and the characteristic equation of linear system in section 2.In section 3,stability analysis and Hopf bifurcation in control model.To verify the theoretic analysis, numerical simulations are given in section 4.Finally, section 5 concludes with some discussion.

2. The characteristic equation of linear system

In article [19]，Xu research a two species Lotka-Volterra model with two delays :

$$\begin{cases} \dot{x}(t) = x(t)[r_1 - a_{11}x(t-\tau_1) - a_{12}y(t-\tau_2)], \\ \dot{y}(t) = y(t)[-r_2 + a_{21}x(t-\tau_2) - a_{22}y(t-\tau_1)]. \end{cases}$$

The dynamical property of the system is given,chen [20] on this basis research the dynamic property of the three species Lotka-Volterra model with two delays.

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1 - a_{11}x_1(t-\tau_1) - a_{13}x_3(t-\tau_2)] \\ \dot{x}_2(t) = x_2(t)[r_2 - a_{22}x_2(t-\tau_1) - a_{23}x_3(t-\tau_2)] \\ x_3(t) = x_3(t)[-r_3 + a_{31}x_1(t-\tau_2) + a_{32}x_2(t-\tau_2) - a_{33}x_3(t-\tau_1)] \end{cases} \tag{2.1}$$

In this paper,the control strategy of the design parameter and the state of feedback control is designed.The Hopf bifurcation of (2.1) system is controlled,make delayed the bifurcation behavior.Parameter perturbation of system (2.1) and increase the time delay control item $\beta x_1(t-\tau_1), \beta x_2(t-\tau_2), \beta x_3(t-\tau_3)$,the control system is:

$$\begin{cases} \dot{x}_1(t) = \alpha x_1(t)[r_1 - a_{11}x_1(t-\tau_1) - a_{13}x_3(t-\tau_2)] + \beta x_1(t-\tau_1) \\ \dot{x}_2(t) = \alpha x_2(t)[r_2 - a_{22}x_2(t-\tau_1) - a_{23}x_3(t-\tau_2)] + \beta x_2(t-\tau_1) \\ x_3(t) = \alpha x_3(t)[-r_3 + a_{31}x_1(t-\tau_2) + a_{32}x_2(t-\tau_2) - a_{33}x_3(t-\tau_1)] + \beta x_3(t-\tau_1) \end{cases} \tag{2.2}$$

The parameter α, β is non negative real number. The positive equilibrium is $E_0(x_1^*, x_2^*, x_3^*)$

$$\begin{cases} x_1^* = \dfrac{\alpha(r_1a_{22}a_{33} - r_2a_{13}a_{32} + r_3a_{22}a_{13} + r_1a_{23}a_{32}) + \beta(a_{22}a_{33} - a_{13}a_{32} + a_{23}a_{32} - a_{22}a_{13})}{\alpha(a_{11}a_{22}a_{33} + a_{22}a_{13}a_{31} + a_{11}a_{23}a_{32})} \\ x_2^* = \dfrac{\alpha(-r_1a_{23}a_{31} + r_2a_{11}a_{33} + r_2a_{13}a_{31} + r_3a_{11}a_{23}) + \beta(-a_{23}a_{31} + a_{11}a_{33} + a_{13}a_{31} - a_{23}a_{11})}{\alpha(a_{11}a_{22}a_{33} + a_{22}a_{13}a_{31} + a_{11}a_{23}a_{32})} \\ x_3^* = \dfrac{\alpha(-r_3a_{11}a_{22} + r_1a_{22}a_{31} + r_2a_{11}a_{32}) + \beta(a_{11}a_{22} + a_{22}a_{31} + a_{11}a_{32})}{\alpha(a_{11}a_{22}a_{33} + a_{22}a_{13}a_{31} + a_{11}a_{23}a_{32})} \end{cases}$$

(H_1)

$$\begin{aligned} & sign(\alpha(a_{11}a_{22}a_{33} + a_{22}a_{13}a_{31} + a_{11}a_{23}a_{32})) \\ &= sign(\alpha(-r_3a_{11}a_{22} + r_1a_{22}a_{31} + r_2a_{11}a_{32}) + \beta(a_{11}a_{22} + a_{22}a_{31} + a_{11}a_{32})) \\ &= sign(\alpha(-r_1a_{23}a_{31} + r_2a_{11}a_{33} + r_2a_{13}a_{31} + r_3a_{11}a_{23}) + \beta(-a_{23}a_{31} + a_{11}a_{33} + a_{13}a_{31} - a_{23}a_{11})) \\ &= sign(\alpha(r_1a_{22}a_{33} - r_2a_{13}a_{32} + r_3a_{22}a_{13} + r_1a_{23}a_{32}) + \beta(a_{22}a_{33} - a_{13}a_{32} + a_{23}a_{32} - a_{22}a_{13})) \end{aligned}$$

The system (2.1)is linearized at positive equilibrium