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Extended rational (G'/G) expansion method for nonlinear partial differential equations

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Abstract. In this article, we use the extended rational (G'/G)- expansion function method to construct exact solutions for some nonlinear partial differential equations in mathematical physics via the (2+1)-dimensional Wu-Zhang equations, the KdV equation, and generalized Hirota–Satsuma coupled KdV equation in terms of the hyperbolic functions, trigonometric functions and rational function, when G satisfies a nonlinear second order ordinary differential equation. When the parameters are taken some special values, the solitary wave are derived from the traveling waves. This method is reliable, simple and gives many new exact solutions.

Keywords: The extended rational(G'/G)- expansion function method, Traveling wave solutions, The (2+1)-dimensional Wu-Zhang equations, The KdV equation, The generalized Hirota–Satsuma coupled KdV equation.

1. Introduction

Nonlinear partial differential equations play an important role in describing the various phenomena not only in physics, but also in biology and chemistry, and several other fields of science and engineering. It is one of the important jobs in the study of the nonlinear partial differential equations are searching for finding the traveling wave solutions. There are many methods for obtaining exact solutions to nonlinear partial differential equations such as the inverse scattering method [1], Hirota's bilinear method [2], Backlund transformation [3], the first integral method [4], Painlevé expansion [5], sine–cosine method [6], homogenous balance method [7], extended trial equation method [8,9], perturbation method [10,11], variation method [12], tanh - function method [13,14], Jacobi elliptic function expansion method [15,16], Exp-function method [17,18] and F-expansion method [19,20] . Wang etal [21] suggested a direct method called the (G'/G) expansion method to find the traveling wave solutions for nonlinear partial differential equations (NPDEs) . Zayed etal [22,23] have used the (G'/G) expansion method and modified (G'/G) expansion method to obtain more than traveling wave solutions for some nonlinear partial differential equations. Shehata [24] have successfully obtained more traveling wave solutions for some important NPDEs when G satisfies a linear differential equations $G'' - \mu G = 0$. In this paper we use the extended rational (G'/G)- expansion function method when G satisfies a nonlinear differential equations $AGG''(\xi) - BGG' - EG^2 - CG'^2 = 0$, where A, B, C, E are real arbitrary constants to find the traveling wave solutions for some nonlinear partial differential equations in mathematical physics namely the (2+1)-dimensional Wu-Zhang equations, the KdV equation and the generalized Hirota–Satsuma coupled KdV equation. We obtain some new kind of traveling wave solutions when the parameter are taking some special values.

2. Description of the extended rational (G'/G) expansion function method for NPDEs

In this part of the manuscript, the extended rational (G'/G) expansion method will be given. In order to apply this method to nonlinear partial differential equations we consider the following steps.
Step 1. We consider the nonlinear partial differential equation, say in two independent variables x and t is given by

$$P(u,u_t,u_x,u_{tt},u_{xx},u_{xt},..)=0, \quad (1)$$

where $u=u(x,t)$ is an unknown function, P is a polynomial in $u=u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved.

Step 2. We use the following travelling wave transformation:

$$u=U(\xi), \qquad \xi=x-kt, \quad (2)$$

where k is a nonzero constant. We can rewrite Eq.(1) in the following form:

$$P(U,U',U'',...)=0 \quad (3)$$

Step 3. We assume that the solutions of Eq. (3) can be expressed in the following form:

$$U(\xi)=\sum_{i=-m}^{m}\frac{a_i\left(G'(\xi_n)/G(\xi_n)\right)^i}{\left[1+\alpha G'(\xi_n)/G(\xi_n)\right]^i}, \quad (4)$$

where $a_i(i=0,\pm1,...,\pm m)$ are arbitrary constants , α is nonzero constant to be determined later, m is a positive integer and $G(\xi)$ satisfies a nonlinear second order differential equation

$$AGG''(\xi)-BGG'-EG^2-CG'^2=0, \quad (5)$$

where A,B,C,E are real nonzero constants.

Step 4. Determine the positive integer m by balancing the highest order nonlinear term(s) and the highest order derivative in Eq (3).

Step 5. Substituting Eq. (4) into (3) along with (5), cleaning the denominator and then setting each coefficient of $(G'(\xi)/G(\xi))^i, i=0,\pm1,\pm2,..$ to be zero, yield a set of algebraic equations for $a_i(i=0,\pm1,...,\pm m)$, k and α .

Step 6. Solving these over-determined system of algebraic equations with the help of Maple software package to determine $a_i(i=0,\pm1,...,\pm m)$, k and α.

Step 7. The general solution of Eq. (5), takes the following cases :

(i) When $B\neq 0$, $B^2+4E(A-C)>0$, we obtain the hyperbolic exact solution of Eq.(5) takes the following form:

$$G(\xi)=\frac{e^{\frac{\xi B}{2(A-C)}}}{[B^2+4E(A-C)]^{\frac{A}{2(A-C)}}}\left[C_1\cosh\left(\frac{\sqrt{B^2+4E(A-C)}}{2A}\xi\right)+C_2\sinh\left(\frac{\sqrt{B^2+4E(A-C)}}{2A-C}\xi\right)\right]^{\frac{A}{(A-C)}} \quad (6)$$

where C_1 and C_2 are arbitrary constants. In this case the ratio between G' and G takes the form

$$\frac{G'}{G}=\frac{B}{2(A-C)}+\frac{\sqrt{B^2+4E(A-C)}}{2(A-C)}\left[\frac{C_1\sinh\left(\frac{\sqrt{B^2+4E(A-C)}}{2A}\xi\right)+C_2\cosh\left(\frac{\sqrt{B^2+4E(A-C)}}{2A}\xi\right)}{C_1\cosh\left(\frac{\sqrt{B^2+4E(A-C)}}{2A}\xi\right)+C_2\sinh\left(\frac{\sqrt{B^2+4E(A-C)}}{2A}\xi\right)}\right] \quad (7)$$

(ii) When $B\neq 0$, $B^2+4E(A-C)<0$, we obtain the trigonometric exact solution of Eq.(5) takes the form

$$\frac{G'}{G}=\frac{B}{2(A-C)}+\frac{\sqrt{-B^2-4E(A-C)]}}{2(A-C)}\left[\frac{-C_1\sin\left(\frac{\xi\sqrt{-B^2-4E(A-C)}}{2A}\right)+C_2\cos\left(\frac{\xi\sqrt{-B^2-4E(A-C)}}{2A}\right)}{C_1\cos\left(\frac{\xi\sqrt{-B^2-4E(A-C)}}{2A}\right)+C_2\sin\left(\frac{\xi\sqrt{-B^2-4E(A-C)}}{2A}\right)}\right] \quad (8)$$