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Finite-time chaos synchronization of the delay hyperchaotic Lü system with uncertain parameters

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Abstract. This paper deals with the finite-time chaos synchronization of the delay hyperchaotic Lü systen with uncertain parameters. Based on the finite time stability theory, a control law is proposed to realize finite-time chaos synchronization for the delay hyperchaotic Lü systen with uncertain parameters. The controller is simple, robust and only part parameters are required to be bounded. Numerical simulation results are given to demonstrate the effectiveness of the proposed finite-time chaos synchronization scheme.

Keywords: Finite-time chaos synchronization; delay hyperchaotic Lü system ; Uncertain parameters.

1. Introduction

Chaos synchronization has attracted a lot of attention from a variety of research fields since the seminal work of Pecora and Carroll [1]. From then on, chaos synchronization has been developed extensively and intensively due to its potential applications in secure communication [2, 3], complex networks [4-7], biotic science [8-14] and so on [15].

Nowadays, different techniques and methods have been put forward to achieve chaos synchronization, for instance, linear and nonlinear feedback synchronization method [16-19], impulsive synchronization method [20-22], tracking synchronization method [23-25], among many others [26-32].

As time goes on, more and more people began to realize the important role of synchronization time. To attain convergence speed, many effective methods have been introduced and finite-time control is one of them. Finite-time synchronization means the optimality in convergence time. Moreover, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties [33-37].

In this paper, we present a controller to realize finite-time synchronization of he delay hyperchaotic Lü systen with uncertain parameters. The controller is robust to parameter uncertainties and simple to be constructed.

2. Prelininary definitions and lemmas

Finite-time synchronization means that the state of the slave system can track the state of the master system after a finite-time. The precise definition of finite-time synchronization is given below.

Definition 1. Consider the following two chaotic systems:

$$\dot{x}_m = f(x_m),$$
$$\dot{x}_s = h(x_m, x_s), \tag{1}$$

where x_m, x_s are two n-dimensional state vectors. The subscripts 'm' and 's' stand for the master and slave systems, respectively. $f: R^n \to R^n$ and $h: R^n \to R^n$ are vector-valued functions. If there exists a constant $T > 0$, such that

$$\lim_{t\to T}\|x_m - x_s\| = 0,$$

and $\|x_m - x_s\| \equiv 0$, if $t \ge T$, then synchronization of the system (1) is achieved in a finite-time.

Lemma 1 [37]**.** Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \le -cV^{\eta}(t),\ \forall t \ge t_0, V(t_0) \ge 0. \tag{2}$$

Where $c > 0, 0 < \eta < 1$ are all constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ t_0 \le t \le t_1, \tag{3}$$

and

$$V(t) \equiv 0, \forall t \ge t_1 \tag{4}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{5}$$

Proof. Consider the following differential equation:

$$\dot{X}(t) = -cX^{\eta}(t),\ X(t_0) = V(t_0). \tag{6}$$

Although this differential equation does not satisfy the global Lipschitz condition, the unique solution of Eq.(6) can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t-t_0). \tag{7}$$

Therefore, form the comparison Lemma, one obtains

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ t_0 \le t \le t_1, \tag{8}$$

and

$$V(t) \equiv 0, \forall t \ge t_1$$

with t_1 given in (5).

Lemma 2. For $a_1, a_2, \ldots, a_n \in R$, the following inequality holds:

$$|a_1| + |a_2| + \cdots + |a_n| \ge \sqrt{a_1^{\ 2} + a_2^{\ 2} + \ldots + a_n^{\ 2}} \tag{9}$$

3. Main results

A chaotic system is extremely sensitive to its initial condition and minor variations of parameters. In actual situation, the system is disturbed by parameter variation which cannot be exactly predicted. The consequence of these uncertainties will destroy the synchronization and even break it. Therefore, it is important and necessary to study the synchronization of systems with uncertainties. In this section, we first discuss finite-time synchronization of the hyperchaotic L ü system. Then , we turn the problem to the system with uncertain parameters.

3.1 chaos synchronization of hyperchaotic L ü system

The delay hyperchaotic L ü system (10) [38]was constructed from the L ü system. The form of the delay hyperchaotic L ü system is given by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1), \\
\dot{x}_2 &= cx_2 - x_1x_3 + x_4(t-\tau), \\
\dot{x}_3 &= x_1x_2 - bx_3, \\
\dot{x}_4 &= -\alpha_1 x_1 - \alpha_2 x_2\ ,
\end{aligned} \tag{10}$$

where a, b, c, α_1, α_2, τ are real positive constants. System (10) is considered as the master system and the slave system is a controlled system as follows:

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + u_1, \\
\dot{y}_2 &= cy_2 - y_1y_3 + y_4(t-\tau) + u_2 \\
\dot{y}_3 &= y_1y_2 - by_3 + u_3 \\
\dot{y}_4 &= -\alpha_1 y_1 - \alpha_2 y_2 + u_4.
\end{aligned} \tag{11}$$

Denote $e_1 = y_1 - x_1, e_2 = y_2 - x_2, e_3 = y_3 - x_3, e_4 = y_4 - x_4$. Subtracting Eq.(10) from Subtracting Eq.(11) we can get the following error system:

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + u_1, \\
\dot{e}_2 &= ce_2 + e_1e_3 - e_1y_3 - y_1e_3 + e_4(t-\tau) + u_2,
\end{aligned}$$