ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 11, No. 2, 2016, *pp*.111-119

Bernoulli Wavelet Based Numerical Method for Solving Fredholm Integral Equations of the Second Kind

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(*Received September 23, 2015, accepted February 20, 2016)*

Abstract. In this paper, a Bernoulli wavelet based numerical method for the solution of Fredholm integral equations of the second kind is proposed. The method is based upon Bernoulli wavelet approximations. The Bernoulli wavelet (BW) is first presented and the resulting Bernoulli wavelet matrices are utilized to reduce the Fredholm integral equations into algebraic equations. Solving these equations using MATLAB to obtain Bernoulli coefficients. The numerical results of the proposed method through the illustrative examples is presented in comparison with the exact and existing methods (Haar wavelet method (HWM) [13], Hermite cubic splines (HCS) [11]) of solution from the literature are shown in tables and figures, which show that the validity and applicability of the technique with higher accuracy even for the smaller values of N.

Keywords: Bernoulli Wavelet, Haar wavelet, Hermite cubic splines, Bernoulli Polynomials, Bernoulli numbers, Fredholm Integral equations.

1. Introduction

Wavelets theory is a relatively new and an emerging tool in applied mathematical research area. It has been applied in a wide range of engineering disciplines; particularly, signal analysis for waveform representation and segmentations, time-frequency analysis and fast algorithms for easy implementation. Wavelets permit the accurate representation of a variety of functions and operators. Moreover, wavelets establish a connection with fast numerical algorithms. Since from 1991 the various types of wavelet method have been applied for numerical solution of different kinds of integral equation, a detailed survey on these papers can be found in [1-6].

Consider the Fredholm integral equation of the second kind:

$$y(t) = f(t) + \int_0^1 K(t,s)\, y(s) ds \quad 0 \le t, s \le 1, \tag{1}$$

where $f(t)$ and the kernels $k(t,s)$ are assumed to be in $L^2(R)$ on the interval $0 \le t, s \le 1.$ We assume that Eq.(1) has a unique solution *y* to be determined. Integral equations find its applications in various fields of science and engineering. There are several numerical methods for approximating the solution of Fredholm integral equations are known and many different basic functions have been used. Such as Galerkin methods for the constructions of orthonormal wavelet bases approached by Liang et.al [7], Maleknejad et al. [8] used the continuous Legendre wavelets, a combination of Hybrid taylor and block-pulse functions [9], Rationalized haar wavelet [10], Hermite Cubic splines [11], Coifman wavelet as scaling functions [12]. Lepik et al. [13] applied the Haar Wavelets, Yousefi et al. [14] have introduced a new CAS wavelet, Babolian et al. [15] derived the operational matrix for the product of two triangular orthogonal functions, Muthuvalu et al. [16] applied Half-sweep arithmetic mean method with composite trapezoidal scheme for the solution of Fredholm integral equations. Keshavarz et al. [17] applied Bernoulli wavelet operational matrix for the approximate solution of fractional order differential equations. In this paper, we introduced the numerical method based on Bernoulli wavelets approximations for solving Fredholm integral equations.

The article is organized as follows: In Section 2, the basic formulation of Bernoulli wavelets and the function approximation is presented. Section 3 is devoted to the method of solution. In section 4, we report our numerical findings and demonstrated the accuracy of the proposed scheme by considering illustrative examples. Conclusion of the proposed method is discussed in section 5.

2. Bernoulli Wavelets and Function Approximation

Bernoulli wavelets are $b_{n,m}(t)=b(k,\hat{n},m,t)$ have four arguments; $\hat{n}=n-1, n=1,2,3,...,2^{k-1}$, k is any positive integer, m is the order of Bernoulli polynomials and t is the normalized time. Then it can be defined [17] on the interval [0, 1) as follows,

$$\mathrm{b}_{m.n}(\mathrm{t}) = \begin{cases} 2^{\frac{k-1}{2}}\tilde{\beta}_m(2^{k-1}t-\hat{n}), \frac{n}{2^{k-1}} \le t < \frac{n}{2^{k-1}} \\ 0, else \end{cases} \tag{2}$$

with

$$\tilde{\beta}_m(t)=\begin{cases} 1, & m=0, \\ \dfrac{1}{\sqrt{\dfrac{(-1)^{m-1}(m!)^2}{(2m)!}\alpha_{2m}}}\beta_m(t), & m>0, \end{cases}$$

where m = 0,1,2,...,M-1 and n = 1,2,...,2^{k-1}. The coefficient $\dfrac{1}{\sqrt{\dfrac{(-1)^{m-1}(m!)^2}{(2m)!}\alpha_{2m}}}$ is for normality, $2^{-(k-1)}$ is the dilation parameter, $\hat{n}2^{-(k-1)}$ is the translation parameter and $\beta_m(t)=\sum_{i=0}^{m}\binom{m}{i}\alpha_{m-i}t^i$ are the well-known Bernoulli polynomials of order m. Where $\alpha_i, i=0,1,...,m$ are Bernoulli numbers. These numbers are a sequence of signed rational numbers which arise in the series expansion of trigonometric functions and can be defined by the identity,

$$\frac{t}{e^t-1}=\sum_{i=0}^{\infty}\alpha_i\frac{t^i}{i!}.$$

The first few Bernoulli numbers are

$$\alpha_0=1,\alpha_1=\frac{-1}{2},\alpha_2=\frac{1}{6},\alpha_4=\frac{-1}{30},\alpha_6=\frac{1}{42},\alpha_8=\frac{-1}{30},\alpha_{10}=\frac{5}{66},\ldots$$

With $\alpha_{2i+1}=0, i=1,2,3,\ldots$

The first few Bernoulli Polynomials are

$$\beta_0(t)=1,\ \beta_1(t)=t-\frac{1}{2},\ \beta_2(t)=t^2-t+\frac{1}{6},$$
$$\beta_3(t)=t^3-\frac{3}{2}t^2+\frac{1}{2}t, \beta_4(t)=t^4-2t^3+t^2-\frac{1}{30},$$
$$\beta_5(t)=t^5-\frac{5}{2}t^4+\frac{5}{3}t^3-\frac{1}{6}t,$$
$$\beta_6(t)=t^6-3t^5+\frac{5}{2}t^4-\frac{1}{2}t^2+\frac{1}{42},\ \ldots$$

The six basis functions are given by:

$$\left.\begin{aligned} b_{10}(t)&=\sqrt{2} \\ b_{11}(t)&=\sqrt{6}\,(4t-1) \\ b_{12}(t)&=\sqrt{10}\,(24t^2-12t+1) \end{aligned}\right\}; \ 0\le t<\frac{1}{2},$$

$$\left.\begin{aligned} b_{20}(t)&=\sqrt{2} \\ b_{21}(t)&=\sqrt{6}\,(4t-3) \\ b_{22}(t)&=\sqrt{10}\,(24t^2-36t+13) \end{aligned}\right\}; \frac{1}{2}\le t<1$$