ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 11, No. 2, 2016, *pp*. 136-145

Haar Wavelet Method for the Numerical Solution of Benjamin–Bona–Mahony Equations

S. C. Shiralashetti[1*],L. M. Angadi[2], A. B. Deshi[1], M. H. Kantli[1]
[1]*P. G. Department of Studies in Mathematics, Karnatak University, Dharwad-580003, India,*
E-mail: shiralashettisc@gmail.com, Mob: +919986323159, Phone: +91836–2215222(O), Fax: +91836–347884
[2]*Govt. First Grade College, Chikodi-591201, India*
(*Received October 30, 2015, accepted February 24, 2016)*

Abstract. In this paper, we proposed an efficient numerical method based on uniform Haar wavelet for the numerical solutions oflinear and nonlinear Benjamin–Bona–Mahony (BBM) Equations. Such types of problems arise in various fields of science and engineering. In present study more accurate solutions have been obtained by Haar wavelet decomposition with multiresolution analysis. Three test problems are considered to check theefficiency and accuracy of the proposed method.An extensiveamount of error analysis has been carried out to obtain the convergence of the method.The numerical results are found in good agreement with exact and finite difference method (FDM), which shows that the solution using Haar wavelet method (HWM) is more effective and accurate and manageable for these equations.

Keywords: Haar wavelet method;Benjamin–Bona–MahonyEquation;Finitedifference method;Numerical simulation; Error analysis.

1. Introduction

Most scientific problems arise in real-world physical problems such as plasma physics, fluid mechanics, solid state physics and in many branches of chemistry [1]. We know that except a limited number of these problems, most of them do not have analytical solutions. The importance of obtaining the approximate solutions of nonlinear partial differential equations in physics and mathematics is still a significant problem that needs new methods to discover approximate solutions. Therefore, these nonlinear equations should be solved using numerical methods i.e. variational iterationmethod (VIM) [4] and homotopy-perturbation method (HPM)[5]. The BBM equation was introduced by Benjamin-Bona-Mahony, as an improvement of the Ko-rtewegde Vries equation (KdV equation). It describes the model for propagation of long waves which incorporates nonlinear and dissipative effects. It is used in the analysis of the surface waves of long wavelength in liquids, hydro magnetic waves in cold plasma, acoustic-gravity waves in compressible fluids, and acoustic waves in harmonic crystals. Many mathematicians paid their attention to the dynamics of the BBM equation.The main mathematical difference between KdV and BBM models can be most readily appreciated by comparing the dispersion relation for the respective linearized equations. It can be easily seen that these relations are comparable only for small wave numbers and they generate drastically different responses to short waves. Recently, various authors have been proposed different methods to solving the different type of BBM equations [6, 13].

In numerical analysis, Wavelets are used as appropriate tools at various places to provide good mathematical model for scientific phenomena, which are usually modeled through linear or nonlinear differential equations.Haar wavelet method is one of them because of Haar functions appearing very attractive in many applications.The previous work in wavelet analysis via Haar wavelets was led by Chen and Hsiao [2], who first derived a Haar operational matrix for the integrals of the Haar function and put the application for the Haar analysis into the dynamic systems. In order to take the advantages of the local property, many authors researched the Haar wavelet to solve the differential and integral equations [7-12].

The objective of the present work is to apply the Haar wavelet method(HWM) for the numerical solution of different types of Benjamin-Bona-Mahony equations and obtained results are compared with the classical FDM and exact solution. The present method is illustrated by some of the Benjamin-Bona-Mahony equations.

The present paper is organized as follows; in section 2, Haar wavelets and its generalized operational matrix of integration are given. Haar Wavelet Method for solving Benjamin-Bona-Mahony equationsis presented in section 3. Section 4 deals with the numerical Experiment, results and error analysis of the illustrative problems. Finally, conclusion of the proposed work is discussed in section 5.

2. Haar wavelets and Operational matrix of integration

We used the simplest wavelet function i.e Haar wavelet. We establish an operational matrix for integration via Haar wavelets.

The scaling function $h_1(x)$ for the family of the Haar wavelets is defined as

$$h_1(x)=\begin{cases}1, & for\ x\in[0,1)\\ 0, & Otherwise\end{cases} \tag{2.1}$$

The Haar wavelet family for $x\in[0,1)$ is defined as

$$h_i(x)=\begin{cases}1, & for\ x\in\left[\frac{k}{m},\frac{k+0.5}{m}\right)\\ -1, & for\ x\in\left[\frac{k+0.5}{m},\frac{k+1}{m}\right)\\ 0, & Otherwise\end{cases} \tag{2.2}$$

In the above definition the integer $m=2^l$, $l=1,2,...,J$, indicates the level of resolution and integer $k=0,1,2,...,m-1$ is the translation parameter. Maximum level of resolution is J. The index i in Eq. (2.2) is calculated using $i=m+k+1$. In case of minimal values $m=1,\ k=0,$ then $i=2$. The maximal value of $i$is $N=2^{J+1}$.

Let us define the grid points $\tilde{x}_j =\ j-0.5)/N,\ J=1,2,...,N,$ discretize the Haar function $h_i(x)$, in this way we get Haar coefficient matrix $H(i,j)=h_i(x_j)$ which has the dimension $N\times N$ .The operational matrix of integration is obtained by integrating (2.2) is as

$$Ph_i(x)=\int_0^x h_i(x)\,dx \tag{2.3}$$

$$Qh_i(x)=\int_0^x Ph_i(x)\,dx \tag{2.4}$$

and

$$Ch_i(x)=\int_0^1 Ph_i(x)\,dx \tag{2.5}$$

These integrals can be evaluated by using equation (2.2) and they are given by

$$Ph_i(x)=\begin{cases}x-\frac{k}{m}, & for\ x\in\left[\frac{k}{m},\frac{k+0.5}{m}\right)\\ \frac{k+1}{m}-x, & for\ x\in\left[\frac{k+0.5}{m},\frac{k+1}{m}\right)\\ 0, & Otherwise\end{cases} \tag{2.6}$$