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Adaptive step forward-backward matching pursuit algorithm

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Abstract. The sparseness adaptive matching pursuit algorithm (SAMP) is a classical algorithm based on compressed sensing theory. Aiming at reconstructing signals with unknown sparsity, an adaptive step forward-backward matching pursuit algorithm (AFBMP) is presented. The AFBMP select matching atoms in the forward processing by using logarithmic variable steps which under the frame of sparseness adaptive matching pursuit algorithm. At the beginning of iterations, high value of step size, causing fast convergence of the algorithm is used to realize the coarse approach of signal sparse, and in the later smaller value of step size is used to realize the precise reconstruction of the sparse signal which equal to half of the previous step. Then AFBMP amend the mistakes which caused in the former stage and delete part of the false atoms in the support set using the backward strategy. Finally it realizes the signal accurately approximate. Experiments show that the AFBMP algorithm can reconstruct the unknown signal more efficiently.

Keywords: Compressed Sensing, Reconstruction Algorithm, Forward-Backward, Sparseness Adaptive, Matching Pursuit Algorithm

1. Introduction

Compressed Sensing theory [1-2] (CS) is a new way to reconstruct signal or image. If the compression ratio is high, the reconstruction error is very small by the CS. In addition, it can compress the data with the data collecting, so the CS can save time and the efficiency is high.

The reconstruction algorithm is the core of the CS and its aim to achieve the original signal based on low dimensional data observed as much as possible. With the development of the CS theory, more and more reconstruction algorithms were proposed in recent years. At present, the common reconstruction algorithm can be divided into three categories: the combinatorial optimization reconstruction algorithm, the convex optimization algorithm and the greedy iterative algorithm. The reconstruction of combinatorial optimization algorithm is good, but not practical. The convex optimization algorithm is too complicated, so that it is not practical too. Such as the Basis pursuit algorithm (BP) [3] is one of the convex optimization algorithm. In recent years, the greedy iterative algorithm is more and more popular, because the accuracy of the reconstruction is high and it is convenient to realize. Such as the Matching Pursuit algorithm(MP)[4],the Orthogonal Matching Pursuit algorithm(OMP)[4-5],the Regularized Orthogonal Matching Pursuit algorithm (ROMP)[6],the Compressive sampling Matching Pursuit algorithm(CoSaMP) [7],the Subspace Pursuit algorithm(SP)[8]and the Sparseness Adaptive Matching Pursuit algorithm(SAMP)[9-10]etc. We all know the linear programming method can ensure the accuracy of the original signal in a certain number of iterations, but the structure of the algorithms is too complicated, so these algorithms can't be widely applied. The greedy iterative algorithm is famous for the fast speed of reconstruction, but we need to know the sparsity of the greedy algorithm in advance. Unfortunately, we can't get the sparsity in practice. In addition, if the sparsity is fixed, the algorithm may affect the precision of the reconstruction.

The Sparseness Adaptive Matching Pursuit algorithm (SAMP) is an improvement algorithm. It breaks through the traditional of the previous algorithm which need to know the sparisity. The algorithm solves the problem of the reconstruction in the case of unknown sparisity for the first time. The reconstruction speed of SAMP mainly depends on the choice of fixed step. If the step size is too large, the reconstruction speed is high and the reconstruction accuracy is low. At the same time, it may cause over estimation. In contrast, if the step size is too small, the reconstruction speed is slower and the reconstruction accuracy is high. Then it may cause under estimation. We all know the SAMP belongs to the forward greedy algorithm and cannot delete redundant atoms. Then some scholars propose some improved algorithms. MSAMP [11] use atom matching test firstly and get the sparsity of the signal. Then it achieves reconstruction in the framework of SAMP. But in the stage of "big step" or "small step", the step value is still a fixed value. So MSAMP may cause over estimation or under estimation. LSAMP [12] use logarithmic variable step and solve the over estimation or under estimation problem by controlling the dual threshold. The reconstruction effect is still good when the sampling rate is low. However, these algorithms are essentially forward greedy algorithms, the biggest drawback of forward greedy algorithm is that the error caused by the previous step iteration can't be modified, once the atom is selected to support set, it will not be deleted. For example, figure 1 supposes that feature vector x is formed by α_1, α_2 which in the observation matrix. While the other vector α_3 is more close to the feature, it will choose α_3 firstly and cannot delete the wrong vector. This result is not what we want. In fact, the forward greedy algorithm is available when the observation matrix is not relevant.

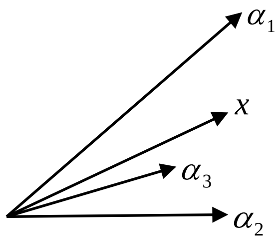

Figure 1

Now we can exploit the backward greedy algorithm to improve the shortcomings. It first selects the relevant atoms and then deletes the atoms one by one for the reconstruction error is smallest. Therefore, according to these two means, an adaptive step forward-backward matching pursuit algorithm is proposed. The new algorithm uses an improved logarithmic variable step in each stage and can delete the redundant atoms. So it can ensure the reconstruction accuracy and overcome the shortage of other algorithms.

Our paper is organized as follows: Section 2 introduces the theory of compressed sensing and reconstruction algorithm. Section 3 introduces the Sparsity Adaptive Matching Pursuit algorithm. Section 4 introduces the adaptive step forward-backward matching pursuit algorithm in detail. Section 5 compares the new algorithm with the common reconstruction algorithm by experiments. Finally, we get the conclusion.

2. The theory of compressed sensing and reconstruction algorithm

The compressed sampling theory pointed out that as long as the signal was sparse, we could reconstruct the signal by low dimensional linear observed matrix.

Suppose x is the original signal and its length is N .The sparsity is K .Set y is the observed signal and its length is M .Let $\Phi_{M\times N}(M<N)$ denotes the measurement matrix. According to the theory of Compressed Sensing, y can be expressed as:

$$y=\Phi x \tag{1}$$

What we want to know is how to reconstruct x form the observed signal y .We usually solve the following optimization problem[13-15]:

$$\min\|x\|_0, \quad s.t. \quad y=\Phi x \tag{2}$$

Obviously, if $M<<N$, equation (1) is a system of indeterminate equations. Then the equations have more than one solution. Equation (2) is a Non-deterministic Polynomial [12]. It is hard to get the solution. But when x is sparse enough and Φ meet the Restricted Isometry Property (RIP) [16-17]:

$$(1-\delta_K)\|x\|_2^2 \le \|\Phi x\|_2^2 \le (1+\delta_K)\|x\|_2^2 \tag{3}$$

Where $\delta_K \in (0,1)$ denotes restricted isometry constant degree and K is the sparsity. Now, solving equation (2) is equivalent to solve l_1 norm minimization problem:

$$\min\|x\|_1, \quad s.t. \quad y=\Phi x \tag{4}$$

The matching pursuit algorithm provides a powerful tool to achieve the solution. OMP algorithm can ensure the result of each iteration is optimal by orthogonal for the selected atoms and reduce the number of iterations. Then one scholar puts forward the regularization orthogonal matching pursuit algorithm. The algorithm selects multiple relates atoms as the candidate set each iteration. Then they use a regularization method to the candidate set and get a new support set. Finally they achieve the reconstruction of atoms. But these methods are based on the sparsity is known. In practical, signal sparsity is often unknown which brings a big challenge to signal reconstruction. So *Thong T.Do* proposed the sparsity adaptive matching pursuit algorithm for the first time and solved the problem of the sparsity was unknown. The algorithm achieves signal reconstruction by fixed step.

3. SAMP