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Haar Wavelet Based Numerical Solution of Elasto-hydrodynamic Lubrication with Line Contact Problems

S.C. Shiralashetti[1]*, M.H. Kantli[1], A. B. Deshi[1]
[1] *Department of Mathematics, Karnatak University, Dharwad,580003, India,*
E-mail:shiralashettisc@gmail.com ;Mobile: +91 9986323159;
Phone: +91 836-2215222(O);Fax: +91 836-347884.
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Abstract. In this paper we present the haar wavelet based numerical solution of the highly nonlinear with coupled differential equation, i. e., elasto-hydrodynamic lubrication with line contact problems. It is a new alternative approach and we explore its perspectives and effectiveness in the analysis of elasto-hydrodynamic lubrication problems. To confirm its versatile features solutions obtained, using haar wavelet based method, are compared with existing method.

Keywords: haar wavelet collocation; elasto-hydrodynamic lubrication; non-linear boundary value problem.

1. Introduction

Wavelet analysis is capable of giving rich and useful description of a function based on a family of basis functions called wavelets. Recently, wavelet analysis has become an important tool in various research areas. The wavelet transform is notable for its ability in time–frequency localization and multi-resolution representation of transient non-stationary signals. Some of the haar wavelet based techniques has been successfully used in various applications such as time–frequency analysis, signal de-noising, non-linear approximation and solving different class of equations arising in fluid dynamics problems (Chen and Hsiao [1], Hsiao and Wang [2], Hsiao [3], Lepik [4-6], Bujurke et al. [7-9] and Islam [10]).

Highly nonlinear and singularity in fluid flow problems is a difficult in numerical simulation. In numerical weather prediction and numerical simulation, the most common methods used are the finite difference method (FDM) on a uniform grid and the spectral method. Since the computational cost of the spectral method is rather large, the FDM is the preferable method at present. The grid space of a uniform grid is restricted to the minimum scale of the synoptic processes concerned. In numerical simulation of a highly nonlinear and singularity, a high resolution is necessary to get a good accuracy. However, this type of problems it is not reasonable to use a fine resolution grid uniformly across the whole domain (the storage and computational cost is very big). To overcome this, it requires the efficient method i.e., haar wavelet method. The main aim of this paper is to present haar wavelet collocation method (HWCM) to solve elasto-hydrodynamic lubrication problems and it has been widely applied in the field of science and engineering numerical simulation.

The present work is organized as follows, in section 2, Wavelet Preliminaries are given. Section 3, discusses about the method of solution. Numerical experiments are presented in section 4. Results and discussions are given in section 5. Finally, conclusion of the proposed work discussed is in section 6.

2. Wavelet preliminaries

2.1. Multi-resolution analysis

The understanding of wavelets is through a multi-resolution analysis. Given a function $f \in L_2(\mathbb{R})$ a multi-resolution analysis (MRA) of $L_2(\mathbb{R})$ produces a sequence of subspaces $V_j, V_{j+1}, \dots$ such that the projections of *f* onto these spaces give finer and finer approximations of the function *f* as $j \to \infty$.

A multi-resolution analysis of $L_2(\mathbb{R})$ is defined as a sequence of closed subspaces $V_j \subset L_2(\mathbb{R}),\ j \in \mathbb{Z}$ with the following properties

(i) $\ldots \subset V_{-1} \subset V_0 \subset V_1 \subset \ldots .$

(ii) The spaces V_j satisfy $\bigcup_{j\in\mathbb{Z}} V_j$ is dense in $L_2(\mathbb{R})$ and $\bigcap_{j\in\mathbb{Z}} V_j = 0$.

(iii) If $f(x) \in V_0$, $f(2^j x) \in V_j$, i.e. the spaces V_j are scaled versions of the central space V_0.

(iv) If $f(x) \in V_0$, $f(2^j x - k) \in V_j$ i.e. all the V_j are invariant under translation.

(v) There exists $\phi \in V_0$ such that $\phi(x-k);\ k \in \mathbb{Z}$ is a Riesz basis in V_0.

The space V_j is used to approximate general functions by defining appropriate projection of these functions onto these spaces. Since the union of all the V_j is dense in $L_2(\mathbb{R})$, so it guarantees that any function in $L_2(\mathbb{R})$ can be approximated arbitrarily close by such projections. As an example the space V_j can be defined like

$$V_j = W_j \oplus V_{j-1} = W_{j-1} \oplus W_{j-2} \oplus V_{j-2} = \ldots = \bigoplus_{j=1}^{J+1} W_j \oplus V_0$$

then the scaling function $h_j(x)$ generates an MRA for the sequence of spaces $\{V_j,\ j \in \mathbb{Z}\}$ by translation and dilation. For each *j* the space W_j serves as the orthogonal complement of V_j in V_{j+1}. The space W_j include all the functions in V_{j+1} that are orthogonal to all those in V_j under some chosen inner product. The set of functions which form basis for the space W_j are called wavelets [10].

2.2. Haar wavelets

The scaling function $h_1(x)$ for the family of the Haar wavelets is defined as

$$h_1(x) = \begin{cases} 1 & for \quad x \in [1,\ 0) \\ 0 & otherwise \end{cases} \tag{1}$$

The Haar wavelet family for $x \in [1,\ 0)$ is defined as

$$h_i(x) = \begin{cases} 1 & for \quad x \in \left[\dfrac{k}{m}, \dfrac{k+0.5}{m}\right) \\ -1 & for \quad x \in \left[\dfrac{k+0.5}{m}, \dfrac{k+1}{m}\right) \\ 0 & otherwise \end{cases} \tag{2}$$

In the above definition the integer, $m = 2^l$, $l = 0, 1, \ldots, J$, indicates the level of resolution of the wavelet and integer $k = 0, 1, \ldots, m-1$ is the translation parameter. Maximum level of resolution is J.

The index i in Eq. (2) is calculated using, $i = m + k + 1$. In case of minimal values $m = 1$, $k = 0$, then $i = 2$. The maximal value of i is $K = 2^{J+1}$. Let us define the collocation points $x_p = \dfrac{p-0.5}{K}$, $p = 1, 2, \ldots, K$, discretize the Haar function $h_i(x)$ and the corresponding Haar coefficient matrix $H(i,\ p) = (h_i(x_p))$, which has the dimension $K \times K$.

The following notations are introduced