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Comparative Study of Finite Element and Haar Wavelet Correlation Method for the Numerical Solution of Parabolic Type Partial Differential Equations

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Abstract. In this paper, we present the comparative study of Haar wavelet collocation method (HWCM) and Finite Element Method (FEM) for the numerical solution of parabolic type partial differential equations such as 1-D singularly perturbed convection-dominated diffusion equation and 2-D Transient heat conduction problems validated against exact solution. The distinguishing feature of HWCM is that it provides a fast converging series of easily computable components. Compared with FEM, this approach needs substantially shorter computational time, at the same time meeting accuracy requirements. It is found that higher accuracy can be attained by increasing the level of Haar wavelets. As Consequences, it avoids more computational costs, minimizes errors and speeds up the convergence, which has been justified in this paper through the error analysis.

Keywords: Haar wavelet collocation method, parabolic equation, Finite difference method, Finite element method, Heat conduction problems.

1. Introduction

Differential equations have numerous applications in many fields such as physics, fluid dynamics and geophysics etc. Many reaction–diffusion problems in biology and chemistry are modeled by partial differential equations (PDEs). These problems have been extensively studied by many authors like Singh and Sharma [1], Giuseppe and Filippo [2] in their literature and their approximate solutions have been accurately computed povided the diffusion coefficients, reaction excitations, initial and boundary conditions are specified in a deterministic way. However, it is not always possible to get the solution in closed form and thus, many numerical methods come into the picture.These are Finite Difference, Spectral, Finite Element and Finite Volume Methods and so on to handle a variety of problems. Many researchers such as Kadalbajoo and Awasti [3],F.De Monte[4] are involved in in developing various numerical schemes for finding solutions of heat conduction problems appear in many areas of engineering and science. So, finding out fiexible techniques for generating the solutions of such PDEs is quite meaningful. Researchers Medvedskii and Sigunov [5] and Doss et.al [6] have used different techniques to compute the above problems and similar ones. Singularly perturbed problemsappear in many branches of engineering, such as fluid mechanics, heat transfer, and problems in structural mechanics posed over thin domains. Theorems that list conditions for the existence and uniqueness of results of such problems are throughly discussed by Ross et.al [7] and Gamel [8].

The application of FEM to various heat conduction problems began through a paper by Zienkienicz and Cheung in 1965 [9]. Subsequently, Wilson and Nickel [10] have studied time dependent FE with variational principle in their work on transient heat conduction problems with Gurtin's Variational principle [11].Zienkienicz and Parekh [12] derived isoparametric finite element formulations for 2-D transient heat conduction problems to approximate the solution in space and time. Argyris et.al [13,14] analyzed structural problems by using real time-space finite elements. A parabolic time-space element, an unconditionally stable in the solution of heat conduction problems through a quasivariational approach was used by Tham and Cheung [15]. Wood and Lewis [16] compared the heat equations for different time-marching schemes. However, it is necessary to choose very small time-steps in order to overcome unwanted numerically induced oscillations in the solution.

From the past few years, wavelets have become very popular in the field of numerical approximations. Among the different wavelet families mathematically most simple are the Haar wavelets. Due to the simplicity, the Haar wavelets are very effective for solving ordinary and partial differential equations. In the previous years, many researchers like Bujurke and Shiralashetti et.al [17,18, and 19] and [67], Hariharan and Kannan[20] have worked with Haar wavelets and their applications. In order to take the advantages of the local property, Chen and Hsiao [21], Lepik [22,23] researched the Haar wavelet to solve the differential and integral equations. Haar wavelet collocation method (HWCM) with far less degrees of freedom and with smaller CPU time provides improved solutions than classical ones, see Islam et.al[24], In the present work, we use FEM and HWCM for solving typical heat conduction problems.

The organization of the present chapter is in the following manner; Haar wavelets and operational matrix of integration in the generalized form are shown in section 2. In section 3 and 4, method of solution of FEM and HWCM are discussed respectively. Section 5 deals with numerical findings with error analysis of the examples. Finally, the conclusion of the proposed work is described in section 6.

2. Haar wavelets and operational matrix of integration

The scaling function $h_1(x)$ for the family of the Haar wavelets is defined as

$$h_1(x) = \begin{cases} 1 & for \ x \in [0,1) \\ 0 & otherwise \end{cases} \tag{2.1}$$

The Haar wavelet family for $x \in [0,1)$ is defined as

$$h_i(x) = \begin{cases} 1 & for \ x \in \left[\frac{k}{m}, \frac{k+0.5}{m}\right) \\ -1 & for \ x \in \left[\frac{k+0.5}{m}, \frac{k+1}{m}\right) \\ 0 & otherwise \end{cases} \tag{2.2}$$

In the definition (2.2), the integer $m = 2^l$, $l = 0,1,...,J$, indicates the level of resolution of the wavelet and integer $k = 0,1,...,m-1$ is the translation parameter. Maximum level of resolution is J . The index i in (2.2) is calculated using $i = m+k+1$. In case of minimal values $m=1,\ k=0$ then $i = 2$. The maximal value of i is $N = 2^{J+1}$.Let us define the collocation points $x_j = \frac{j-0.5}{N}$, $j = 1,2,...,N$, discretize the Haar function $h_i(x)$, in this way, we get Haar coefficient matrix $H(i,j) = h_i(x_j)$ which has the dimension $N \times N$. For instance, $J = 3 \Rightarrow N = 16$, then we have

$$\mathrm{H}(16,16) = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{pmatrix}$$

The operational matrix of integration via Haar wavelets is obtained by integrating (2.2) is as,