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# Adomian decomposition method for fuzzy differential equations with linear differential operator

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**Abstract.** In this paper we have taken the fuzzy differential equation with linear differential operator. We have used Adomian decomposition method (ADM) to find the approximate solution. We have given several numerical examples and by comparing the numerical results obtain from ADM with the exact solution, we have studied their accuracy.

**Keywords:** fuzzy differential; fuzzy differential equations; adomian decomposition method.

## 1. Introduction

Fuzzy differential equations are very useful to model dynamical systems whose uncertainty is characterized by a non-random process [9]. So the existence and uniqueness of the fuzzy differential equation is a area of great interest and this have been studied in [10, 13, 14, 19, 21, 23]. Various kind of fuzzy differential equation and their application have been studied by many researchers. Bede et al. interpret first order linear fuzzy differential equations by using the strongly generalized differentiability concept [11]. Chalco-Cano, Roman-Flores study the class of first order fuzzy differential equations where the dynamics is given by a continuous fuzzy mapping which is obtain via Zadeh's extension principle [12]. Solutions of first order fuzzy differential equations have been also studied in [15, 17, 18, 20, 22, 24, 25, 26]. An extension of differential transformation method using the concept of generalized H-differentiability has been studied by Allahviranloo et al. in [16].

The Adomian Decomposion Method (ADM) was first introduced by Adomian in 1980 [1]. ADM is very powerful tool to solve algebraic, differential, integral and integro-differential equations involving non-linear functional [2, 3, 4, 5]. A second-order fuzzy differential equation has been solved by using Adomian method under strongly generalized differentiability in [8]. Using ADM hybrid fuzzy differential equations have been solved in [7]. Numerical approximation of fuzzy first-order initial value problem by using ADM is presented in [6]. In this paper, we develop numerical method for fuzzy differential equations with linear differential operator by an application of the ADM. The structure of this paper is organized as follows: Section 2 contains some basic definitions of fuzzy sets and fuzzy number. Section 3 contains solution procedure of fuzzy differential equations with linear differential operator by ADM. In section 4, the proposed method is illustrated by two numerical examples and we compare ADM solution with exact solution to check the accuracy of the method. Finally the conclusion and future research is given in section 5.

## 2. Preliminaries

**Definition 2.1.** If $X$ is a collection of objects denoted by $x$ then a fuzzy set $\tilde{A}$ in $X$ is a set of ordered pairs denoted and defined by:

$\tilde{A}=\{(x,\mu_{\tilde{A}}(x))/x\in X\}$, were $\mu_{\tilde{A}}(x)$ is called membership function or grade of membership (also degree of compatibility or degree of truth) of $x$ in $\tilde{A}$ which maps $X$ to $[0,1]$.

**Definition 2.2.** $\alpha$-cut of a fuzzy $\tilde{A}$ set is a crisp set $A_\alpha$ and defined by

$A_\alpha=\{x/\mu_{\tilde{A}}(x)\geq\alpha\}$, where $\tilde{A}=\{(x,\mu_{\tilde{A}}(x))\}$.

**Definition 2.3.** A fuzzy set $\tilde{A}$ is said to be convex fuzzy set if $A_\alpha$ is a convex set for all $\alpha\in(0,1]$.

**Definition 2.4.** A fuzzy set $\tilde{A}$ is said to be normal fuzzy set if there exist an element $(a,1)\in\tilde{A}$.

**Definition 2.5.** If a fuzzy set is convex, normalized and its membership function, defined in $\mathbb{R}$, is piecewise continuous then it is called as fuzzy number.

A triangular fuzzy number $\tilde{A}$ is denoted by $(a_1,\ a_2,\ a_3)$ and it is a fuzzy set $\{(x, \mu_{\tilde{A}}(x))\}$ where

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0,\ \text{otherwise} \end{cases}.$$

$\tilde{A}$ is called positive triangular fuzzy number if $a_1 > 0$ and negative triangular fuzzy number if $a_3 < 0$.

**Definition 2.6.** [27]. Let E be the set of all upper semicontinuous normal convex fuzzy numbers with bounded $\alpha$-cut intervals. It means if $\tilde{v} \in$ E then the $\alpha$-cutset is a closed bounded interval which is denoted by

$$v_{\ \alpha} = [v_1, v_2].$$

For arbitrary $u_{\ \alpha} = [u_1, u_2]$,$v_{\ \alpha} = [v_1, v_2]$and $k \ge 0$, addition $(u_{\ \alpha} + v_{\ \alpha})$ and multiplication by $k$ are defined as $(u+v)_1(\alpha) = u_1(\alpha) + v_1(\alpha)$, $(u+v)_2(\alpha) = u_2(\alpha) + v_2(\alpha)$, $(ku)_1(\alpha) = ku_1(\alpha)$, $(ku)_2(\alpha) = ku_2(\alpha)$.

Since each $y$ ∈R can be regarded as a fuzzy number $\tilde{y}$ defined by

$$\mu_{\tilde{y}}(x) = \begin{cases} 1 & if\ \ x = y, \\ 0 & if\ x \ne y. \end{cases}$$

The Hausdorff distance between fuzzy numbers given by D :E×E→ $R_+ \cup \{0\}$,

$$\mathrm{D}(\tilde{u}, \tilde{v}) = \sup_{\alpha \in [0,1]} \mathrm{Max}\{|u_1(\alpha) - v_1(\alpha)|, |u_2(\alpha) - v_2(\alpha)|\}.$$

It is easy to see that D is a metric in E and has the following properties (see [28])

(i) $\mathrm{D}(\tilde{u} \oplus \tilde{w}, \tilde{v} \oplus \tilde{w}) = \mathrm{D}(\tilde{u}, \tilde{v}), \forall\ \tilde{u}, \tilde{v}, \tilde{w} \in$E,

(ii) $\mathrm{D}(k \odot \tilde{u}, k \odot \tilde{v}) = |k|\mathrm{D}(\tilde{u}, \tilde{v}), \forall k \in$R, $\tilde{u}, \tilde{v} \in$E,

(iii) $\mathrm{D}(\tilde{u} \oplus \tilde{v}, \tilde{w} \oplus \tilde{e}) \le \mathrm{D}(\tilde{u}, \tilde{w}) + \mathrm{D}(\tilde{v}, \tilde{e}), \forall\ \tilde{u}, \tilde{v}, \tilde{w}, \tilde{e} \in$E,

(iv) (D,E) is a complete metric space.

**Definition 2.7.** (see [24]). Let $f$:R →E be a fuzzy valued function. If for arbitrary fixed $t_0$ ∈R and $\epsilon > 0$, a $\delta > 0$ such that

$$|t - t_0| < \delta \Longrightarrow \mathrm{D}(f(t), f(t_0)) < \epsilon,$$

$f$ is said to be continuous.

## 3. Fuzzy differential equation with linear differential operator

The fuzzy differential equation with linear differential operator is as follows:

$$L\tilde{u}(t) + R\tilde{u}(t) + N(t, \tilde{u}(t)) = \tilde{g}(t) \tag{1}$$

$\tilde{u}^j(a) = \tilde{a}_j, j = 0, 1, \ldots\ldots, m-1$, where $\tilde{g}(t)$ is a fuzzy function of $t$, $L$ is the highest order linear differential operator of order m, $R$ is the remaining part of the linear differential operator and $N$ may be linear or nonlinear function of $t$ and $\tilde{u}(t)$. Here, in general, we take $N$ as a nonlinear function of $t$ and $\tilde{u}(t)$ such that

$$N_1(t, u_1, u_2) = \sum_{i=1}^{l_1} F_{1i}(t, u_1) F_{2i}(t, u_2) \tag{2}$$

$$N_2(t, u_1, u_2) = \sum_{j=1}^{l_2} G_{1j}(t, u_1) G_{2j}(t, u_2) \tag{3}$$

where $u_\alpha(t) = [u_1(t, \alpha), u_2(t, \alpha)], [N_1(t, u_1, u_2), N_2(t, u_1, u_2)] = N(t, [u_1(t, \alpha), u_2(t, \alpha)])$,

$F_{1i}(t, u_1)$, $G_{1j}(t, u_1)$ are functions of $t$ and $u_1(t, \alpha)$ and $F_{2i}(t, u_1)$, $G_{2j}(t, u_1)$ are functions of $t$ and $u_2(t, \alpha)$, i = $1, 2, \ldots., l_1$, $j = 1, 2, \ldots\ldots, l_2$.

Let, $[H_1(u_1, u_2), H_2(u_1, u_2)] = R[u_1(t, \alpha), u_2(t, \alpha)]$, $a_{j\alpha} = [a_{j1}(\alpha),\ a_{j1}(\alpha)]$ and

$$g_\alpha(t) = [g_1(t, \alpha), g_2(t, \alpha)] \tag{4}$$

Now, from (1) we get

$$L[u_1(t, \alpha), u_2(t, \alpha)] + R[u_1(t, \alpha), u_2(t, \alpha)] + N(t, [u_1(t, \alpha), u_2(t, \alpha)]) = [g_1(t, \alpha), g_2(t, \alpha)] \tag{5}$$

Therefore,

$$Lu_1(t, \alpha) + H_1(u_1, u_2) + N_1(t, u_1, u_2) = g_1(t, \alpha) \tag{6}$$

and

$$Lu_2(t, \alpha) + H_2(u_1, u_2) + N_2(t, u_1, u_2) = g_2(t, \alpha) \tag{7}$$