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# Approximate Solutions to System of Nonlinear Partial Differential Equations using Reduced Differential Transform Method

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**Abstract.** The main objective of this paper is to use the reduced differential transform method (RDTM) for finding the analytical approximate solutions for solving systems of nonlinear partial differential equations (NPDEs). The approximate solutions obtained by RDTM is verified by comparison with the exact solutions to show that the RDTM is quite accurate, reliable and can be applied for many other nonlinear partial differential equations. The method considers the use of the appropriate initial or boundary conditions and finds the solution without any discretization, transformation, or restrictive assumptions. This method is a simple and efficient method for solving the nonlinear partial differential equations. The numerical results show that this method is a powerful tool for solving systems of NPDEs. The analysis shows that our analytical approximate solutions converge very rapidly to the exact solutions.

**Keywords:** reduced differential transforms method, nonlinear partial differential equations, analytic and approximate solutions.

## 1. Introduction

Nonlinear partial differential equations (NPDEs) are mathematical models that are used to describe complex phenomena arising in the world around us. The nonlinear equations appear in many applications of science and engineering such as fluid dynamics, plasma physics, hydrodynamics, solid state physics, optical fibers, acoustics and other disciplines [1]. On the other hand, there are many effective methods for obtaining the anaylatic approximate solutions and exact solutions of NPDEs samong of these methods are the inverse scattering method [2], Hirota's bilinear method [3], Backlund transformation [4, 5], Painlevé expansion [6] sine-cosine method [7], homogenous balance method [8], homotopy perturbation method [9-12], variation method [13, 14], Adomian decomposition method [15, 16], perturbation method [17, 18], tanh-function method [19-21], Jacobi elliptic function expansion method [22-25], F-expansion method [26, 27], (G'/G)-expansion method [28-32], exp-function method [33-35], complex transformation [36 ] and Riccati equations method [37, 38 ]. Recently Mabood [39] and Mohamed et al. [40-42] used the optimal homotopy asymptotic to study the MHD slips flow over radiating sheet with heat transfer, the flow heat transfer viscoelastic fluid in an axisymetric channel with a porous wall and for the heat transfer in hollow sphere with the Robin boundary conditions. Also, Mabood et al. [43] have discussed the analytical solutions for radiation effects on heat transfer in Blasius flow.

In the present article, we use the reduced differential to transform method (RDTM) which discussed in [44-47], to construct an appropriate solution of some highly nonlinear partial differential equations of mathematical physics. The reduced differential transforms technique is an iterative procedure for obtaining a Taylor series solution of differential equations. This method reduces the size of computational work and easily applicable to many nonlinear physical problems. In this paper, we discuss the analytic approximate solution for two systems of nonlinear wave equations, these systems can be seen in [48, 49]. In mathematical physics, they play a major role in various fields, such as plasma physics, fluid mechanics, optical fibers, solid state physics, geochemistry and so on.

$$u_t - u_{xxx} - 2vu_x - u\,v_x = 0, \tag{1}$$

and

$$\upsilon_t - u\,u_x = 0. \tag{2}$$

One of these systems is the generalized coupled Hirota Satsuma KdV system given by

$$u_t - \frac{1}{2}u_{xxx} + 3uu_x - 3\frac{\partial}{\partial x}(vw) = 0, \tag{3}$$

$$\upsilon_t + \upsilon_{xxx} - 3u\upsilon_x = 0, \tag{4}$$

$$w_t + w_{xxx} - 3u\,w_x = 0. \tag{5}$$

The paper has been organized as follows. Notations and basic definitions are given in Section 2. In Section 3, we apply the RDTM to solve two types of NPDEs. Conclusions are given in Section 4.

## 2. Preliminaries and notations

In this section, we give some basic definitions and properties of the reduced differential transform method which are further used in this paper. Consider a function of three variables $u(x,y,t)$ and suppose that it can be represented as a product of three single-variable functions, i.e., $u(x,y,t) = f(x)h(y)g(t)$. Based on the properties the (2+1) of-dimensional differential transform, the function $u(x,y,t)$ can be represented as follows:

$$u(x,y,t) = \left(\sum_{i=0}^{\infty} F(i)x^i\right)\left(\sum_{j=0}^{\infty} H(j)x^j\right)\left(\sum_{l=0}^{\infty} G(l)x^l\right) = \sum_{k=0}^{\infty} U_k(x,y)t^k \tag{6}$$

where $U_k(x,y)$ is called $t$-dimensional spectrum function of $u(x,y,t)$. The basic definitions of RDTM are introduced as follows [44-47].

**Definition 2.1** If the function $u(x,y,t)$ is analytic and differentiated continuously with respect to time $t$ and space in the domain of interest, then let

$$U_k(x,y) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,y,t)\right]_{t=0}, \tag{7}$$

where the $t-$ dimensional spectrum function $U_k(x,y)$ is the transformed function. In this paper the lowercase $u(x,y,t)$ represents the original function while the uppercase $U_k(x,y)$ stands for the transform function.

**Definition 2.2** The differential inverse transform $U_k(x,y)$ is defined as follows:

$$u(x,y,t) = \sum_{k=0}^{\infty} U_k(x,y)t^k. \tag{8}$$

Then, combining Eqs. (7) and (8) we have:

$$u(x,y,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,y,t)}{\partial t^k}\right]_{t=0} t^k. \tag{9}$$

From the above definitions, it can be found that the concept of the RDTM is derived from the power series expansion. To illustrate the basic concepts of the RDTM, consider the following nonlinear partial differential equation written in an operator form:

$$L[u(x,y,t)] + R[u(x,y,t)] + N[u(x,y,t)] = g(x,y,t) \tag{10}$$

with initial condition

$$u(x,y,0) = f(x,y), \tag{11}$$

where $L = \frac{\partial}{\partial t}$, $R$ is a linear operator which has partial derivatives, $N$ is a nonlinear operator and $g(x,y,t)$ is an inhomogeneous term. According to the RDTM, we can construct the following iteration formula:

$$(k+1)U_{k+1}(x,y,t) = G_k(x,y) - R[U_k(x,y)] - N[U_k(x,y)] \tag{12}$$

where $U_k(x,y)$, $R[U_k(x,y)]$, $N[U_k(x,y)]$ and $G_k(x,y)$ are the transformations of the functions $u(x,y,t)$, $R[u(x,y,t)]$, $N[u(x,y,t)]$ and $g(x,y,t)$ respectively. From the initial condition (8), we write:

$$U_0(x,y) = f(x,y). \tag{13}$$