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Analysis of a delayed predator-prey system with Holling type-IV functional response and impulsive diffusion between two patches

Xingping Zhu [1], Xuerong Shi [2]
[1] *Yancheng Higher Normal School, Yancheng, 224005, China, E-mail: sxryctc@163.com.*
[2] *School of Mathematics and statistics, Yancheng Teachers University, Yancheng, 224002, China, E-mail: sxryctc@163.com.*
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Abstract. Due to the extensive existence of time delay for natural population, it is necessary to take the effect of time delay into account in forming a biologically meaningful mathematical model. In view of this, a delayed predator-prey system with Holling type-IV functional response and impulsive dispersal between two patches is formulated. By using comparison theorem of impulsive differential equation and some analysis techniques, we obtain a predator-extinction periodic solution, which is globally attractive. Furthermore, it is proved that the investigated system is permanent. Numerical simulations are carried out to illustrate the theoretical results.

Keywords: predator-prey; time-delay; impulsive dispersal; global attractivity; permanence.

1. Introduction

Dispersal is a ubiquitous phenomenon in natural world. Its importance in understanding the ecological and evolutionary dynamics of populations was mirrored by a large number of mathematical models devoted to it in the scientific literature. The persistence and extinction for ordinary differential equation and delayed differential equation models were investigated[1-3]. Global stability of equilibrium and periodic solution for diffusing models were studied[4-6].

However, in all of above population dispersing systems, it is always assumed that the dispersal occurs at every time. For example, in [3], Huang proposed the following periodic predator-prey system with Holling-IV functional response:

$$\begin{cases} \dot{x}_1 = x_1\left[b_1(t) - a_1(t)x_1 - \frac{c_1(t)y}{e(t)+\beta(t)x_1+x_1^2}\right] + D(t)(x_2 - x_1). \\ \dot{x}_2 = x_2[b_2(t) - a_2(t)x_2] + D(t)(x_1 - x_2). \\ \dot{y} = y\left[-d(t) + \frac{c_2(t)x_1}{e(t)+\beta(t)x_1+x_1^2} - q(t)y\right]. \end{cases} \tag{1.1}$$

The function $\frac{c(t)x_1(t)}{e(t)+\beta(t)x_1(t)+x_1^2(t)}$ represents the functional response of predator to the prey in patch 1. Let $\psi(t, x_1(t)) = \frac{c(t)x_1(t)}{e(t)+\beta(t)x_1(t)+x_1^2(t)}$, then we have

$$\frac{\partial}{\partial x_1}\psi(t, x_1(t)) \geq 0, 0 < x_1(t) \leq \sqrt{e(t)},$$
$$\frac{\partial}{\partial x_1}\psi(t, x_1(t)) < 0, x_1(t) > \sqrt{e(t)}.$$

In practice, it is often the case that diffusion occurs at certain moment. For example, when winter comes, birds will migrate between patches in search for a better environment, whereas they do not diffuse in other seasons, and the excursion of foliage seeds occurs at certain moment every year. Therefore, it is not reasonable to characterize the population movements in these cases with continuous dispersal models. This short-time scale dispersal is more appropriately assumed to be in the form of impulses in the modeling process. With the developments and applications of impulsive differential equations, theories of impulsive differential equations have been introduced into population dynamics, and many important studies have been performed [7-11]. Hui [8] proposed the following single model with impulsive diffusion:

$$\begin{cases} x_1'(t) = x_1(t)\big(a_1 - b_1 x_1(t)\big), & \\ x_2'(t) = x_2(t)\big(a_2 - b_2 x_2(t)\big), & t \neq n\tau, \\ \Delta x_1(t) = d_1\big(x_2(t) - x_1(t)\big), & t = n\tau, \\ \Delta x_2(t) = -d_2\big(x_2(t) - x_1(t)\big), & \end{cases} \tag{1.2}$$

where a_i, b_i (i = 1, 2) are the intrinsic growth and density-dependent parameters of the population in the *i* th patch, d_i is the net dispersal rate between the *i* th patch and *j* th patch ($i \neq j$, i, j = 1, 2). $\Delta x_i(t) = x_i(n\tau^+) - x_i(n\tau)$, where $x_i(n\tau^+)$ represents the density of the population in the *i* th patch immediately after the *n* th diffusion pulse at time $t = n\tau$, $x_i(n\tau)$ represents the density of the population in the *i*th patch before the *n*th diffusion pulse at time $t = n\tau$,($n = 1, 2,\cdots$, i = 1, 2).

It is well known that the time delay is quite common for natural population. In order to reflect the dynamical behaviors of models that depend on the past history of system, it is necessary to take time delay into account in forming a biologically mathematical model. Delay differential equations have attracted a significant interest in recent years due to their frequent appearance in a wide range of applications, which serve as mathematical models describing various phenomena in physics, biology, physiology, and engineering, see [12-16] and references therein, their research topics include global asymptotic stability of the equilibria, existence of periodic solutions, complicated behaviors and chaos.

Motivated by above analysis, in this paper, we will consider a delayed predator-prey system with Holling type-IV functional response and impulsive diffusion between two patches:

$$\begin{cases} x_1'(t) = x_1(t)[a_1 - b_1 x_1(t) - \frac{c_1 y(t)}{e + x_1(t) + x_1^2(t)}], & \\ x_2'(t) = x_2(t)\big(a_2 - b_2 x_2(t)\big), & t \neq n\tau, \\ y'(t) = y(t)[-d + \frac{c_1 x_1(t-\tau_1)}{e + x_1(t-\tau_1) + x_1^2(t-\tau_1)} - qy(t-\tau_2)], & \\ \Delta x_1(t) = d_1\big(x_2(t) - x_1(t)\big), & t = n\tau, \\ \Delta x_2(t) = -d_2\big(x_2(t) - x_1(t)\big), & \\ \Delta y(t) = 0, & \end{cases} \tag{1.3}$$

with initial conditions

$$x_1(s) = \phi_1(s), x_2(s) = \phi_2(s), y(s) = \phi_3(s),$$
$$\phi = (\phi_1, \phi_2, \phi_3)^T \in C([-\tilde{\tau}, 0], R_+^3), \phi_i(0) > 0, i = 1,2,3.$$

In this case, we suppose that the system is composed of two patches connected by diffusion and occupied by a single species. x_i (i = 1, 2) denotes the density of prey species in the *i*th patch, respectively, and *y* is the density of predator species. a_i and b_i denote the intrinsic growth rate and the density dependence rate of prey species in patch *i*(i = 1, 2), d is the death rate of the predator, and q represents the density dependence rate of predator species in patch 1, c_1 is the capturing rate of the predator, c_2/c_1 is the conversion rate of the nutrient into the production rate of the predator. $\tilde{\tau} = \max\{\tau_1, \tau_2\}$, $\tau_1 \geq 0$ is a constant delay due to the gestation of the predator. In addition, we have included the term $-qy(t-\tau_2)$ in the dynamics of predator y to incorporate the negative feedback of predator crowding. d_i is the net dispersal rate between the *i*th patch and *j*th patch ($i \neq j$, i, j = 1, 2), $0 < d_i < 1$ for i = 1, 2.

Other part of this paper is organized as follows. Some important Lemmas are presented in section. In section 3, the global attractively of the predator-extinction periodic solution and permanence of system (1.3) are investigated. In section 4, some numerical simulations are presented to illustrate the feasibility of our results. In the last section, we give a brief discussion of our results.

2. Preliminaries

In this section, we will give some definitions and lemmas.

Let $R_+ = [0, +\infty)$, $R_+^3 = \{x \in R^3, x \geq 0\}$, the map $f = (f_1, f_2, f_3)^T$ is defined by the right-hand sides of the first three equations of system (1.3), suppose $V: R_+ \times R_+^3 \to R_+$, then V is said to belong to V_0 if

(1) V is continuous in $[n\tau, (n+1)\tau] \times R_+^3$, and, for each $x \in R_+^3$, $n \in N$, $\lim_{(t,y)\to(n\tau^+,x)} V(t,y) = V(n\tau^+, x)$ exists.

(2) V is locally Lipschitzian in x.

Definition 2.1. Let $V \in V_0$, then for $(t, x) \in [n\tau, (n+1)\tau] \times R_+^3$, the upper right derivative of $V(t, x)$ with respect to the impulsive differential equation (1.3) is defined as