

Synchronization of the neurons with external disturbance via single sliding mode controller

Zuolei Wang^{1*}, Xingping Zhu², Xuerong Shi¹, Wen Yun¹, Liying Zheng¹

¹ School of Mathematics and Statistics, Yancheng Teachers University,
Yancheng, 224002, China, E-mail: wzlyctc@163.com.

² Yancheng Higher Normal School, Yancheng, 224005, China.

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Abstract. The synchronization of the Hindmarsh-Rose neuron system with external disturbance is investigated via sliding mode control. Based on the Lyapunov stability theory, a single sliding mode controller is derived even when the response Hindmarsh-Rose neuron system with external disturbance. The numerical simulation is presented to verify the effectiveness of the proposed control scheme.

Keywords: synchronization; hindmarsh-rose system; chaotic bursting; sliding mode control.

1. Introduction

The brain consists of about a hundred specialized modules with different functions. These modules form complex networks [1] which shows complicated dynamic behaviors, some of which are closely related to physiological phenomena of the brain. Therefore, many researchers have explored the dynamics between the neurons of the brain to explain some neurophysiologic phenomena[2-6], especially the synchronization of the neurons [7-12]. Experimental studies [2,8] have pointed out that the synchronization is significant in the information processing of large ensembles of neurons. Hence, it is necessary to employ networks to investigate the complex spatial-temporal behavior of neural systems. Some results have been obtained about it. For example, in [13], it was noted in three types of regular networks that the critical values depended on specific coupling styles when neurons achieved complete synchronization. Shi et al. studied spike synchronization and burst synchronization of two coupled Hindmarsh-Rose (HR) neurons[14]. Wang et al. investigated ordered burst synchronization and complex spatial temporal firing behavior in a ring network of MHH neurons with excitatory chemical synapses[15]. Zheng et al. investigated the effect of various network parameters on bursting dynamics in a small-world HR neuronal network in detail[16].

In this paper, the synchronization of two identical Hindmarsh-Rose (HR) neuron systems is to be investigated. Based on the Lyapunov stability theory, a single sliding mode controller is derived even when the response Hindmarsh-Rose neuron system with external disturbance. Furthermore, the sufficient conditions for synchronization of the coupled systems with chaotic bursting behavior can be obtained. Finally, numerical simulations are given to verify the effectiveness of the proposed scheme.

2. Hindmarsh-Rose neuron system

In this section, the considered Hindmarsh-Rose neuron systems is given to realize the synchronization of the neurons with external disturbance. The HR neuronal model was first proposed by Hindmarsh and Rose as a mathematical representation of the firing behavior of neurons, and it was originally introduced to give a bursting type with long interspike intervals of real neurons [17]. The form of Hindmarsh-Rose system is given by

$$\begin{aligned}\dot{x}_1 &= ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext}, \\ \dot{x}_2 &= c - dx_1^2 - x_2, \\ \dot{x}_3 &= r(S(x_1 + k) - x_3),\end{aligned}\tag{1}$$

where x_1 is the membrane potential, x_2 is associated with the fast current of Na^+ or K^+ ions, and x_3 is associated with the slow current of, for example, Ca^+ ions. $a, b, c, d, r, S, k, I_{ext}$ are real constants. The Hindmarsh-Rose system is a slow-fast system. Slow oscillations x_3 drive the fast subsystem (x_1, x_2)

through periods of oscillatory and quiescent behavior. Model (1) may describe regular bursting or chaotic bursting for certain domains of the parameters. If the parameters are taken as $a = 3.0$, $b = 1.0$, $c = 1.0$, $d = 5.0$, $r = 0.006$, $S = 4.0$, $k = 1.6$, system (1) is regular bursting for $I_{ext} = 2.0$ and chaotic bursting for $I_{ext} = 3.0$, respectively (see Fig.1).

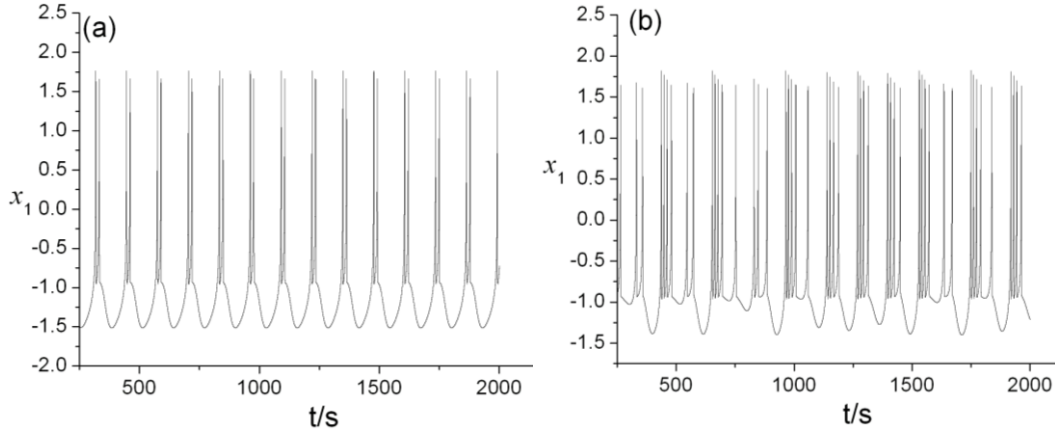


Fig.1. (a) Regular bursting of system (1) for $I_{ext} = 2.0$,

(b) Chaotic bursting of system (1) for $I_{ext} = 3.0$.

The response Hindmarsh-Rose system with external disturbances is described as follows:

$$\begin{aligned}\dot{y}_1 &= ay_1^2 - by_1^3 + y_2 - y_3 + I_{ext} + d_1(t) + U_1, \\ \dot{y}_2 &= c - dy_1^2 - y_2 + d_2(t), \\ \dot{y}_3 &= r(S(y_1 + k) - y_3) + d_3(t),\end{aligned}\quad (2)$$

where $d_1(t)$ and $d_3(t)$ are mismatched disturbances and $d_2(t)$ is a matched disturbance [18,19]. It is assumed that these disturbances are bounded, i.e. $\|d_i(t)\| \leq \sigma < 1 (i=1,2,3)$, where σ is a constant. The aim of this paper is to design a controller U_1 such that the response Hindmarsh-Rose system (2) with disturbances is synchronous with the drive Hindmarsh-Rose system (1).

For this end, let the error variables as

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \quad e_3 = y_3 - x_3, \quad (3)$$

in view of systems (1) and (2), we have the following error dynamics:

$$\begin{aligned}\dot{e}_1 &= a(y_1 + x_1)e_1 - b(y_1^2 + y_1x_1 + x_1^2)e_1 + e_2 - e_3 + d_1(t) + U_1, \\ \dot{e}_2 &= -d(y_1 + x_1)e_1 - e_2 + d_2(t), \\ \dot{e}_3 &= rSe_1 - re_3 + d_3(t).\end{aligned}\quad (4)$$

3. Synchronization of Hindmarsh-Rose systems with disturbances

3.1. Switching surface and controller design

In this section, sliding mode control method is used to synchronize the coupled Hindmarsh-Rose systems with perturbations. This method involves two basic steps. The first step is selecting an appropriate switching surface such that the sliding motion on the sliding manifold is stable and ensures $\lim_{t \rightarrow \infty} e_i = 0 (i=1,2,3)$. The second step is establishing a robust control law which guarantees the existence of the sliding manifold $s_1(t) = 0$ even in the event of external perturbation.

Let

$$U_1 = u_1 - e_1 - e_2 - a(y_1 + x_1)e_1 + b(y_1^2 + y_1x_1 + x_1^2)e_1, \quad (5)$$

and we have the following error system: