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System of linear equations in imprecise environment

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Abstract. The paper discusses linear systems in imprecise environment. In this paper we have developed the solution procedure of system of linear equations with coefficients and the right-hand side as fuzzy and intuitionistic fuzzy numbers. We have also calculated the necessary and sufficient conditions for the existence of the solutions by developing some theorems. We have solved the system by using the concept of Strong and Weak solution with numerical example.

Keywords: fuzzy system of linear equation (FSLE); intuitionistic fuzzy system of linear equation (IFSLE), trapezoidal fuzzy number (TrFN); trapezoidal intuitionisic fuzzy number (TrIFN); strong and weak solutions.

1. Introduction

There are so many applications of systems of linear equations in various areas of mathematical, physical and engineering sciences such as traffic flow, circuit analysis, heat transport, structural mechanics, fluid flow etc. In most of the applications, the system's parameters and measurements are vague or imprecise. In that situation we can represent the systems with given data as fuzzy and more generally Intuitionistic fuzzy numbers rather than crisp numbers.

Fuzzy linear systems are the linear systems whose parameters are all or partially represented by fuzzy numbers. A general model for solving a Fuzzy linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number was first proposed by Friedman et al. [12]. They have used the parametric form of fuzzy numbers and replace the original $n \times n$ fuzzy system by a $2n \times 2n$ crisp system. Fuzzy linear system has been studied by several authors [1,2,4,13,14,15] but there are no such papers on intuitionistic fuzzy linear systems.

In this paper we have developed an approach to solve system of linear equations in imprecise environment i.e. fuzzy and intuitionistic fuzzy environment following Friedman et al. [12]. In Section-3 we have discussed the detailed solution procedure of system of linear equations by using the concept of Strong and Weak solution. We have also illustrated the method by considering a numerical example where coefficients are taken as TrIFNs.

2. Preliminaries

Definition 2.1: Fuzzy Set: Let X be a universal set. The fuzzy set $\tilde{A} \subseteq X$ is defined by the set of tuples as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : \mu_{\tilde{A}}: X \to [0,1]\}$.

The membership function $\mu_{\tilde{A}}(x)$ of a fuzzy set $\tilde{A}$ is a function with mapping $\mu_{\tilde{A}}: X \to [0,1]$. So every element x in X has membership degree $\mu_{\tilde{A}}(x)$ in [0,1] which is a real number.

Definition 2.2:**$\propto$-Level or $\propto$-cut of a fuzzy set:** Let X be an universal set. Let $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\} (\subseteq X)$ be a fuzzy set. $\propto$ -cut of the fuzzy set $\tilde{A}$ is a crisp set. It is denoted by $A_\propto$. It is defined as

$$A_\propto = \{x : \mu_{\tilde{A}}(x) \geq \propto \ \forall x \in X\}$$

Definition-2.3: Intuitionistic Fuzzy Sets: Let $U = \{x_1, x_2, \dots, x_n\}$ be a finite universal set. An Intuitionistic Fuzzy Set $\tilde{A}^i$ in a given universal set U is an object having the form

$$\tilde{A}^i = \{\langle x_i, \mu_{\tilde{A}^i}(x_i), \upsilon_{\tilde{A}^i}(x_i)\rangle : x_i \in U\}$$

Where the functions

$\mu_{\tilde{A}^i}: U \to [0,1]$; i.e. , $x_i \in U \to \mu_{\tilde{A}^i}(x_i) \in [0,1]$

and $\upsilon_{\tilde{A}^i}: U \to [0,1]$; i.e. , $x_i \in U \to \upsilon_{\tilde{A}^i}(x_i) \in [0,1]$

define the degree of membership and the degree of non-membership of an element $x_i \in U$, such that they satisfy the following conditions:

$$0 \le \mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i) \le 1, \forall x_i \in U$$

which is known as Intuitionistic Condition. The degree of acceptance $\mu_{\tilde{A}^i}(x_i)$and of non-acceptance $\upsilon_{\tilde{A}^i}(x_i)$ can be arbitrary.

Definition-2.4: $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-cuts: A set of (α, β)-cut, generated by IFS $\tilde{A}^i$, where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \le 1$ is defined as

$$\tilde{A}^i{}_{\alpha,\beta} = \begin{cases} \left(x, \mu_{\tilde{A}^i}(x), \upsilon_{\tilde{A}^i}(x)\right); & x \in U \\ \mu_{\tilde{A}^i}(x) \ge \alpha, \upsilon_{\tilde{A}^i}(x) \le \beta; & \alpha, \beta \in [0,1] \end{cases},$$

where (α, β)-cut, denoted by $\tilde{A}^i{}_{\alpha,\beta}$, is defined as the crisp set of elements x which belong to $\tilde{A}^i$ at least to the degree α and which does belong to $\tilde{A}^i$ at most to the degree β.

Definition 2.5: Fuzzy Number: $\tilde{A} \in \mathcal{F}(R)$ is called a fuzzy number where R denotes the set of whole real numbers if

i. $\tilde{A}$ is normal i.e. $x_0 \in R$ exists such that $\mu_{\tilde{A}}(x_0) = 1$.

ii. $\forall \propto \in (0,1]$ $A_\propto$ is a closed interval.

If $\tilde{A}$ is a fuzzy number then $\tilde{A}$ is a convex fuzzy set and if $\mu_{\tilde{A}}(x_0) = 1$ then $\mu_{\tilde{A}}(x)$ is non decreasing for $x \le x_0$ and non increasing for $x \ge x_0$.

Definition-2.6: Intuitionistic Fuzzy Number (IFN): An Intuitionistic Fuuzy Number $\tilde{A}^i$ is

i. An Intuitionistic Fuuzy Subset on the real line

ii. Normal i.e. there exists at least one $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}^i}(x_0) = 1$(so $\upsilon_{\tilde{A}^i}(x_0) = 0$)

iii. Convex for the membership function $\mu_{\tilde{A}^i}$ i.e.

$$\mu_{\tilde{A}^i}\big(\lambda x_1 + (1 - \lambda x_2)\big) \ge \min\{\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\}; \forall\, x_1, x_2 \in \mathbb{R}\ , \lambda \in [0,1]$$

iv. Concave for the non-membership function $\upsilon_{\tilde{A}^i}$ i.e.

$$\upsilon_{\tilde{A}^i}\big(\lambda x_1 + (1 - \lambda x_2)\big) \le \max\{\upsilon_{\tilde{A}^i}(x_1), \upsilon_{\tilde{A}^i}(x_2)\}; \forall\, x_1, x_2 \in \mathbb{R}\ , \lambda \in [0,1]$$

Definition-2.7: Trapezoidal Intuitionistic Fuzzy Number: A Trapezoidal Intuitionistic Fuzzy Number (TrIFN) is denoted by $\tilde{A}^i = \langle (a_1, a_2, a_3, a_4), (a_1{}', a_2, a_3, a_4{}') \rangle$ is a special Intuitionistic Fuzzy Set on a real number set $\mathbb{R}$, whose membership function and non-membership function are defined as

$$\mu_{\tilde{A}^i}(x) = \begin{cases} 0 & x \le a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1\ , & a_2 \le x \le a_3\ , \\ \frac{a_4 - x}{a_4 - a_3}\ , & a_3 \le x \le a_4 \\ 0\ , & a_4 \le x \end{cases}$$

$$\upsilon_{\tilde{A}^i}(x) = \begin{cases} 1 & x \le a_1{}' \\ 1 - \dfrac{x - a_1{}'}{a_2 - a_1{}'}, & a_1{}' \le x \le a_2 \\ 0\ , & a_2 \le x \le a_3 \\ 1 - \dfrac{a_4{}' - x}{a_4{}' - a_3}\ , & a_3 \le x \le a_4{}' \\ 1\ , & a_4{}' \le x \end{cases}$$

and $a_1{}' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4{}'$

Note:-

1. If $a_2 = a_3$ then Trapezoidal Intuitionistic Fuzzy Number (TrIFN) is transformed into Triangular Intuitionistic Fuzzy Number (TIFN) $(\langle (a_1, a_2, a_4), (a_1{}', a_2, a_4{}') \rangle)$.

2. If $a_1{}' = a_1 \le a_2 \le a_3 \le a_4 = a_4{}'$ and then Trapezoidal Intuitionistic Fuzzy Number (TrIFN) is transformed into Trapezoidal Fuzzy Number (TrFN) (a_1, a_2, a_3, a_4).

Definition-2.8: The $m \times n$ linear system

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= y_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= y_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= y_n \end{aligned} \tag{2.1}$$