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Pinning hybrid synchronization of time-delay hyperchaotic Lü systems via single linear control

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Abstract. The pinning hybrid synchronization of time-delay hyperchaotic Lü systems is investigated via linear control. Based on Lyapunov stability theory, the coexistence of pinning anti-synchronization and complete synchronization of time-delay hyperchaotic Lü systems is obtained via one single controller. Sufficient conditions are obtained to achieve the hybrid Pinning synchronization. Numerical simulations are presented to demonstrate the effectiveness of the proposed schemes.

Keywords: pinning hybrid synchronization; time-delay; hyperchaotic system; linear control.

1. Introduction

During the last decades, synchronization of chaotic systems, an important topic in nonlinear science, has attracted more and more attention and has been explored intensively. Many kinds of synchronizations, such as complete synchronization [1], phase synchronization [2], generalized synchronization [3], lag synchronization [4], Q-S synchronization [5], anti-synchronization [6], time-scale synchronization [7], functional synchronization [8], projective synchronization [9], cluster synchronization [10], etc., have been proposed and successfully applied to the chaos synchronization of chaotic or hyperchaotic systems.

Many theoretical studies of chaos synchronization have been carried out for coupled systems[11,12]. In most cases of drive-response synchronization, all the states of the response system synchronize to the corresponding states of drive system in terms of the same synchronization regime. As far as we know, complete synchronization is characterized by the equality of state variables while evolving in time. Anti-synchronization is characterized by the disappearance of the sum of relevant variables [5]. Does the phenomenon that some states of the interactive systems are synchronized in terms of one type of synchronization regime, and other states synchronized in terms of another type exist in unidirectionally and linearly coupled chaotic systems? There is no doubt that it is an interesting problem. Therefore, inspired by [13,14], it is invited to investigate the coexistence synchronization problems of time delay chaotic systems by using single control input both for theoretical research and practical applications. However, up to our knowledge, there have been few (if any) results of an investigation for the delay chaotic systems via a single controller with one variable in the literature, especially the time-delay hyperchaotic systems.

In this paper, we will show that the pinning hybrid synchronization of time-delay hyperchaotic Lü systems can be achieved only by a single variable controller. Other parts of this paper are arranged as follows. In section 2, dynamical behavior of time delay Hyperchaotic system is explored and 2D overview hyperchaotic attractors are given. In section 3, schemes to achieve the pinning hybrid synchronization are proposed. Section 4 demonstrates the numerical simulations to verify the theoretical results. Some conclusions are drawn in section 4.

2. Dynamical behavior of time delay Hyperchaotic system

In this paper, the considered hyperchaotic system with time delay is described as

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= cy - xz + w(t-\tau), \\ \dot{z} &= xy - bz, \\ \dot{w} &= -\alpha_1 x - \alpha_2 y, \end{aligned} \tag{1}$$

where $\tau > 0$ is the time delay. When $\tau = 0$, system (1) is the hyperchaotic system constructed from the Lü system by Pang S, Liu Y in [15]. For convenience, we call it delay Hyperchaotic Lü system. When a = 35,

b = 3, c = 20, α_1 = 2, α_2 = 2 and the time delay τ is chosen as 1, by the Galerkin approximation technique, an algorithms considered by Ghosh D, Chowdhury R, Saha P for calculating Lyapunov exponents for system with time delay[15], system (1) has two positive Lyapunov exponents, i.e., λ_1= 1.4523, λ_2 = 0.3562, and the hyperchaotic attractors of system (1) are shown in Fig. 1 (2D overview).

Furthermore, in real application, smaller number of controller and simpler form of controller are practical greatly. In following sections, we will investigate the coexistence of pinning anti- synchronization and complete synchronization of time-delay hyperchaotic 4D systems via simple feedback controller.

3. Pinning Hybrid synchronization of time-delay hyperchaotic systems

In this section, we describe the synchronization effects in large spatially ordered ensembles of oscillators, i.e., the systems are arranged in a regular spatial structure. The simplest example is a chain, where each element interacts with its nearest neighbors, if the first and last elements of the chain are also coupled, then it becomes a ring structure.

In the multiple structure coupled in a ring with hyperchaotic 4D systems, only the second variable of each chaotic system interacts with its nearest neighbors, the first and last systems of the chain are also coupled, which can be given as the following forms:

$$\begin{aligned} \dot{x}_i &= a(y_i - x_i), \\ \dot{y}_i &= cy_i - x_i z_i + w_i(t-\tau) + \rho\, v_i, \\ \dot{z}_i &= x_i y_i - bz_i, \\ \dot{w}_i &= -\alpha_1 x_i - \alpha_2 y_i, \end{aligned} \tag{2}$$

where $i = 1, 2, ..., n, v_i = (y_{i+1} - y_i)\ (i \neq n), v_n = (y_1 - y_n)$ and ρ is the coupling coefficient.

We choose the states x, y, z and w of an isolated node dynamical system (1) as a synchronous solution of the controlled complex dynamical network (2) because it is a diffusive coupling network. The target of this paper is to find a single controller, which makes the states variable x_i, y_i and w_i in response system pinning anti-synchronize x, y and w in drive system, respectively, while the third state variable z_i in response system is pinning complete -synchronized to z in drive system. For this purpose, let

$$e_{ix} = x_i + x,\ e_{iy} = y_i + y,\ e_{iz} = z_i - z,\ e_{iw} = w_i + w. \tag{3}$$

The aim is to propose simple input controller such that the state errors in (3) satisfy

$$\lim_{t\to\infty} e_{ix}(t) = 0,\ \lim_{t\to\infty} e_{iy}(t) = 0,\ \lim_{t\to\infty} e_{iz}(t) = 0,\ \lim_{t\to\infty} e_{iw}(t) = 0. \tag{4}$$

Firstly, we will show the coexistence of anti-synchronization and complete-synchronization via single linear controller. The controller are imposed on the second formula to change the dynamics of the nonlinear response system as shown in Eq. (2). The result is given in **Theorem 1**.

Theorem 1. For suitable value of ε, the state variables x_i, y_i and w_i in the following controlled complex dynamical network (5) are anti-synchronized to the synchronous solutions x, y and w, while the third state variable z_i in (5) is complete-synchronized to z.

$$\begin{aligned} \dot{x}_i &= a(y_i - x_i), \\ \dot{y}_i &= cy_i - x_i z_i + w_i(t-\tau) + \rho\, v_i + u_i, \\ \dot{z}_i &= x_i y_i - bz_i, \\ \dot{w}_i &= -\alpha_1 x_i - \alpha_2 y_i, \end{aligned} \tag{5}$$

where $u_1 = -\varepsilon e_{1y}$, u_i =0 ($i = 2, 3, ..., n$).

Proof. The error system of (5) and (1) can be governed by the following dynamical system

$$\begin{aligned} \dot{e}_{ix} &= a(e_{iy} - e_{ix}), \\ \dot{e}_{iy} &= ce_{iy} - z_i e_{ix} + xe_{iz} + e_{iw}(t-\tau) + \rho\, \vartheta_i + u_i, \\ \dot{e}_{iz} &= y_i e_{ix} - xe_{iy} - be_{iz}, \end{aligned}$$