

Haar wavelet collocation method to solve problems arising in induction motor

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Abstract. In this paper, Haar wavelet collocation method has been employed to solve problems occur in the mathematical modeling of induction motor. To approximate the solution algorithm based on Haar wavelet is considered. The order of convergence is estimated for discussed problems. The accuracy issues of the solutions are demonstrated by comparing with other numerical techniques existing in the literature.

Keywords: Haar wavelets; induction motor; collocation method; convergence analysis.

1. Introduction

Alfred Haar [1] introduced the notion of wavelets which are called Haar wavelets. These wavelets placed a crucial role for the numerical solution of differential or integral equations. At present there are two approaches to applying the Haar wavelet for integrating ordinary differential equations (ODE). In case of the first method for integrating ODE concept of operational matrix is introduced by Chen and Hsiao [2, 3]. Another approach is called direct method due to Lepik [4]. In this approach Haar functions are integrated directly. The direct method is easily applicable for calculating integrals of arbitrary order but the operational matrix method has been used mainly for first order integrals. Haar wavelets consists of piecewise constant functions and are therefore the simplest orthogonal wavelets with compact support. Here the fact that Haar wavelets are not continuous and hence derivatives do not exists at the braking points. So that it is not possible to apply the Haar wavelets for solving ODEs directly. The main advantage of the method is that it can be used directly without using restrictive assumptions.

The fifth-order differential equations arise in modeling of induction motor with two rotor circuits. This model contains two stator state variables, two rotor state variables and one shaft speed. Normally, two more variables must be added to account for the effects of a second rotor circuit representing deep bars, a starting cage or rotor distributed parameters. To avoid the computational burden of additional state variables when additional rotor circuits are required model is often limited to the fifth order and rotor impedance is algebraically altered as function of rotor speed. This is done under the assumption that the frequency of rotor currents depends on rotor speed. This approach is efficient for the steady state response with sinusoidal voltage [5].

The existence and uniqueness solutions of fifth order boundary value problems (BVPs) are discussed by Agarwal [6]. Over the years some researchers have worked on induction motor problems by using different methods for numerical solutions. Reddy et al. [7] have demonstrated the superiority of the HWCMM for the solution of seventh order ODEs of induction motor with two rotor circuits. Siddiqi et al. [8] estimated the solution for linear special case fifth-order two-point boundary value problems by non-polynomial sextic spline method (NPSS). Farajeyan and Jalilian [9] have found the numerical solution by fifth order BVPs in off step points (OSPM). Mehdi Golomi et al. [10] solved fifth order differential equations by He's Variational iteration method (VIM). This paper deals with fifth order ODEs arising in modeling of induction motor; these problems have the following general representation:

$$y^{(5)}(x) = f(x, y, y^{(1)}, y^{(2)}, y^{(3)}, y^{(4)}), \quad x \in (c, d), \quad (1)$$

subject to the following conditions:

Case I: Initial value problem:

$$y(c) = \alpha_1, y^{(1)}(c) = \beta_1, y^{(2)}(c) = \gamma_1, y^{(3)}(c) = \delta_1, y^{(4)}(c) = \eta_1. \quad (2)$$

Case II: Boundary value problems of **Type 1:**

$$y(c) = \alpha_2, y^{(1)}(c) = \beta_2, y^{(2)}(c) = \gamma_2, y(d) = \delta_2, y^{(1)}(d) = \eta_2. \quad (3)$$

Case III: Boundary value problem of **Type 2:**

$$y(c) = \alpha_3, y^{(1)}(c) = \beta_3, y^{(3)}(c) = \gamma_3, y(d) = \delta_3, y^{(1)}(d) = \eta_3. \quad (4)$$

Where α_i 's, β_i 's, γ_i 's, δ_i 's, η_i 's, c and d are real constants for $i = 1, 2, 3$.

This article is organized as, in section 2 notations of Haar wavelets and their integrals are introduced. In section 3, numerical algorithm based on wavelets is introduced. In section 4 convergence analysis is presented. In section 5 we reported our numerical results with comparison. In the final section conclusion of our work has been discussed.

2. Haar wavelets and their integrals

In this section, we obtain orthogonal basis for the subspaces of $L^2[c, d]$ called Haar wavelet family. For this notations introduced in Ref. [4] are used. The interval $[c, d]$ is divided into 2^{J+1} subintervals of equal length $\left(\Delta t = \frac{(d-c)}{2^{J+1}}\right)$, where J is called maximal level of resolution. We have coarser resolution values $j = 0, 1, 2, \dots, J$ and translation parameter $k = 0, 1, 2, \dots, 2^j - 1$. With these two parameters i^{th} Haar wavelet in Haar family is defined as

$$h_i(t) = \begin{cases} 1, & \text{for } t \in [\zeta_1(i), \zeta_2(i)), \\ -1, & \text{for } t \in [\zeta_2(i), \zeta_3(i)), \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

here $i = m + k + 1$, $\zeta_1(i) = c + 2k\mu\Delta t$, $\zeta_2(i) = c + (2k+1)\mu\Delta t$ and $\zeta_3(i) = c + 2(k+1)\mu\Delta t$, where $\mu = 2^{J-j}$.

Above equations are valid for $i > 2$. $h_1(t)$ and $h_2(t)$ are called father and mother wavelets in Haar wavelet family and are given by

$$h_1(t) = \begin{cases} 1, & \text{for } t \in [c, d), \\ 0, & \text{otherwise,} \end{cases} \quad (6)$$

$$h_2(t) = \begin{cases} 1, & \text{for } t \in [c, p), \\ -1, & \text{for } t \in [p, d), \\ 0, & \text{otherwise,} \end{cases} \quad (7)$$

where, $p = \frac{c+d}{2}$.

Any function which is having finite energy on $[c, d]$, i.e. $f \in L^2[c, d]$ can be decomposed as infinite sum of Haar wavelets:

$$f(x) = \sum_{i=1}^{\infty} a_i h_i(x), \quad (8)$$

where a_i 's are called Haar coefficients. If f is either piecewise constant or wish to approximate by piecewise constant on each subinterval then the above infinite series will be terminated at a finite number of terms. Since, we have explicit expression for each member of Haar family (5- 7). We can integrate as many times depend upon the context. The following notations are used for γ times of integration of members in the family defined on $[c, d]$:

$$P_{\gamma,i}(t) = \int_c^t \int_c^t \dots \int_c^t h_i(x) dx^\gamma, \quad (9)$$